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STUDY OF FLOWS AROUND SPUR-DIKES

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ABSTRACT

Flow patterns around spur-dikes are very complicated,especially three-dimensional flows with secondary flows downstream of spur-dikes. The presently used two-dimensional numerical analyses do not estimate downstream flows from spur-dikes precisely,because of inaccurate estimates of the effects of secondary flows. A quasi three-dimensional model(a two and a half dimensional model) was developed, independent of presently used methods of numerical analysis. this three-dimensional model enabled the estimation of the effects of secondary flows,which are difficult to estimate with the two-dimensional models presently used.

INTRODUCTION

An important aspect of river planning is to precisely estimate flow and sediment transport as wells as river bed variations around river structures like spur-dikes. Generally, sand deposition behind structures is caused by sediment transport due to turbulence and secondary flows. Two-dimensional (plane or vertical two-dimensional) diffusion of turbulence behind obstructions in the flow path has been studied by Plandtl.(1979), and the structure of flows behind spur-dikes has been investigated by Kakizaki and Hasegawa(1992). Based on this research, Muneta(1992).(1992),(1993) has developed a model incorporating the influence of turbulent diffusion into calculations of depth averaged flows behind spur-dikes. This has shown that momentum diffusion due to turbulence plays an important role in determining average flow velocities where differences of Reynolds Numbers in the transverse direction are very large, such large differences are common behind river structures. In addition to deoth averaged flows, however, secondary flows are active behind structures, and accurate calculations of secondary flows are indispensable to determine sediment and river bed variations. This study will investigate a method to estimate secondary flows behind structures with numerical analysis. Shimizu(1991). developed a three-dimensional numerical model to directly estimate three-dimensional flows behind structures. This method is considered very accurate because the equations of motion and the continuity equation for three-dimensional flows are directly calculated. However, it has not been developed for practical use because the calculations are difficult and require extensive calculation time. This study aims to develop a model which simply yields accurate results comparable to the three-dimensional numerical analyses but with calculation times similar to a two-dimensional model.

Purpose of Study

To determine the basic structure of flows behind structures, three-dimensional flows around a single spur-dike were measured with laser current meters. Figure-1 shows experimental results of transverse flow velocity distributions, [vertical flow velocity distributions at (a). (b). and (c) on a line 1cm outside and downstream from the end of a spur-dike.]

(See Figure-2 and Table-1). (Flows away from the spur-dike are considered negative.) The figure shows characteristics of secondary flows behind the spur-dike, flows toward the spur-dike are large near the river bed, while flows away from the spur-dike are otherwise large. Near the river bed these flows may play an important role in sand deposition. A two-dimensional calculation method cannot estimate this phenomenon, while the application of a three-dimensional model may enable calculations of secondary flows. This study develops a quasi-three-dimensional model which practically and economically calculates secondary flows by substituting the depth averaged velocities U and V (U downward and V transverse) obtained from the two-dimensional shallow water flow model into the equation of motion in the three-dimensional downward and transverse directions.

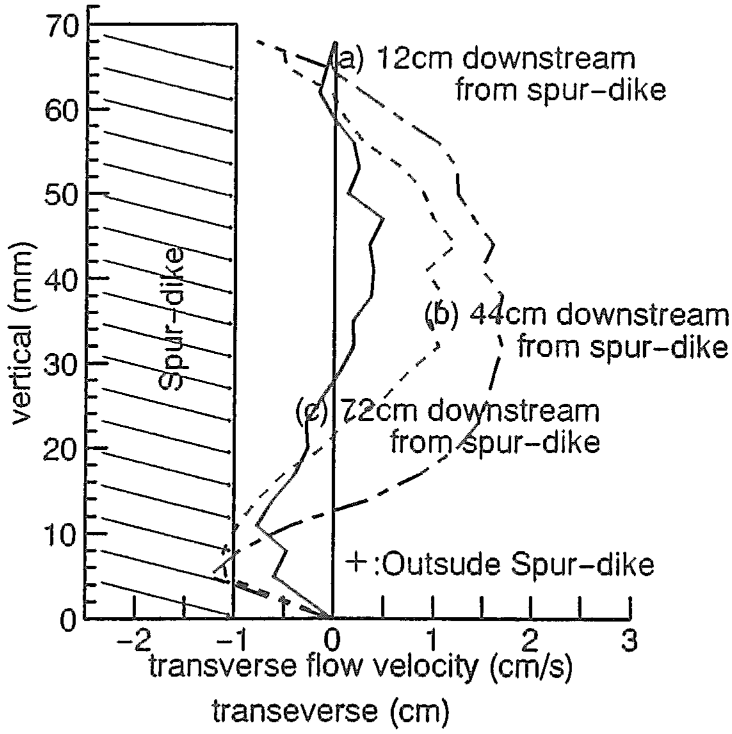

figure-1 transverse flow velocity below spur-dike

Basic Concepts of The Model

Engelund (1985) developed a method to estimate secondary flows in a channel with uniform curvature. This method uses the following equation obtained by omitting inertial terms other than centrifugal forces, and advection diffusion terms except in the z-direction from the equation of motion in the transverse direction.

$$-\frac{u_s^2}{r} + \frac{1}{\rho}\frac{\partial p}{\partial n} = \frac{\partial}{\partial z}(K_*\frac{\partial v_n}{\partial z}) \tag{1}$$

Here s_o,n,and$z$ are distances in the downward, transverse, and vertical directions; u_s and v_n are flow velocity components in the $s_o$and n directions; ρ is the density of fluid; p is the pressure; $K_*$is $\kappa u_* h/6$ (k= Karman constant, u_* = friction velocity, h= water depth); and r is the radius of curvature of the streamlines. A flow velocity distribution for secondary flows around a curvature is calculated by integrating Equation (1) twice with a slip velocty near the river bed and zero shear stress at the water surface as boundary conditions. Expansion of this method into calculations of secondary flows behind spur-dikes is effected as follows: Based on the depth averaged flow velocities determined from the two-dimensional shallow water model, a logarithmic velocity distribution is assumed as the first approximation [Equation (8) below] and substituted into the equation of three-dimensional motion.this is then transformed into the form of Equation(1). Next the equation is integrated twice with respect to z, giving flow velocity distributions in the vertical direction. In addition, the flow velocity distributions are averaged over the water depth. The results and are then used to determine the surface slope.

Deduction of Basic Equations

The three-dimensional equations of motion in the transverse direction in a Cartesian co-ordinate system are expressed by (2) and (3) :

$$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial y} + \frac{1}{\rho}(\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \tau_{yy}}{\partial y} + \frac{\partial \tau_{zy}}{\partial z}) \tag{2}$$

and therefore,

$$u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial y}+\frac{\partial}{\partial x}(\nu_t\frac{\partial v}{\partial y})+2\frac{\partial}{\partial y}(\nu_t\frac{\partial v}{\partial y})+\frac{\partial}{\partial z}(\nu_t\frac{\partial v}{\partial z})+\frac{\partial}{\partial x}(\nu_t\frac{\partial u}{\partial y}) \quad (3)$$

here x,y,and$z$ are the co-ordinate axes in the downward, transverse, and vertical directions,u,v,and$w$ are the flow velocity components in the x,y,and$z$ axis directions,τ_{xy} ,τ_{yy},andτ_{zy} are the shear stress components, and$\nu_t$is the eddy viscosity coefficient. The following description considers the river bed $z=0$ for convenience. This model assumes the flow velocity in the vertical direction (z) $w\approx 0$, and omits terms with w in Equations (2) and (3). The equations of motion in the transverse direction, omitting w, are

$$u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial y}+\frac{\partial}{\partial x}(\nu_t\frac{\partial v}{\partial y})+2\frac{\partial}{\partial y}(\nu_t\frac{\partial v}{\partial y})+\frac{\partial}{\partial z}(\nu_t\frac{\partial v}{\partial z})+\frac{\partial}{\partial x}(\nu_t\frac{\partial u}{\partial y}) \quad (4)$$

here

$$\tau_{xy}=\nu_t(\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}),\quad \tau_{yy}=2\nu_t(\frac{\partial v}{\partial y}),\quad \tau_{yz}=\nu_t(\frac{\partial v}{\partial z}) \quad (5)$$

It is assumed that the eddy viscosity coefficient, can be expressed in term of acomponent which is constant in the vertical direction dependent on water depth only and a component expressing the vertical distribution:

$$\nu_t=K_0k(\xi) \quad (6)$$

here K_0 , the component with respect to water depth, is κu_*h To obtain an accurate flow velocity distribution under uniform flow conditions, above and below 20% water depth, $\kappa\xi$ is expressed by the following equations as in Ratteray-Mitsuda (1974)

$$k(\xi)=\begin{cases}\alpha\xi(1-\xi) & \xi<0.2\\ \alpha/\beta & \xi\geq 0.2\end{cases} \quad (7)$$

here $\alpha=6.25,\beta=6$
Assuming one-dimensional uniform flow, the relation between the average flow velocities $U$and$V$, and the individual flow velocities u and$v$ are expressed by:

$$u=U\Phi(\xi_0,\xi),\qquad v=V\Phi(\xi_0,\xi) \quad (8)$$

here $\Phi(\xi_0,\xi)$is the dimensionless flow velocity distribution of the one-dimensional uniform flow [logarithmic below 20% of water depth and parabolic above], obtained by substituting (7) into (6); ξ is the dimensionless vertical direction ($\xi=z/h$), $\xi_0$is the dimensionless distance from the river bed at a point with flow velocity=0, and $U$and$V$ are average water depth flow velocities in the x and y directions determined by the twodimensional shallow water flow model. Equation (9) is obtained by substituting (8) into (4).

$$\begin{aligned}\Phi(\xi_0,\xi)^2 & \left[U\frac{\partial V}{\partial x}+V\frac{\partial V}{\partial y}\right]=-g\frac{\partial(H+h')}{\partial y}\\ &+ k(\xi)\Phi(\xi_0,\xi)\left[\frac{\partial}{\partial x}(K_0\frac{\partial U}{\partial y})+\frac{\partial}{\partial x}(K_0\frac{\partial V}{\partial x})+2\frac{\partial}{\partial y}(K_0\frac{\partial V}{\partial y})\right]+\frac{K_0}{h\partial\xi}(k(\xi)\frac{\partial v}{h\partial\xi})\end{aligned} \quad (9)$$

here H is the water elevation, calculated from the two-dimensional shallow water flow model, and h is the water level balanced by the parts outside the logarithmic and parabolic parts. The calculation of $(H+h')$ enables an estimate of the effect secondary flows. Equation (10) is obtained by substituting (9) into the second order equation of $\Phi(\xi_0,\xi)$

$$a_v\Phi(\xi_0,\xi)^2+b_vk(\xi)\Phi(\xi_0,\xi)+c_v=\frac{\partial}{\partial\xi}\left(k(\xi)\frac{\partial v}{\partial\xi}\right) \quad (10)$$

here the coefficients of $\Phi(\xi_0,\xi)^2$ and $\Phi(\xi_0,\xi)$ are a_v,and b_v,and the term of the pressure gradient is c_v .

$$a_v = \underbrace{\frac{h^2}{K_0}(U\frac{\partial V}{\partial x}+V\frac{\partial V}{\partial y})}_{Acceleration\ term} \tag{11}$$

$$b_v = \underbrace{-\frac{h^2}{K_0}\left[\frac{\partial}{\partial x}(K_0\frac{\partial U}{\partial y})+\frac{\partial}{\partial x}(K_0\frac{\partial V}{\partial x})+2\frac{\partial}{\partial y}(K_0\frac{\partial V}{\partial y})\right]}_{Diffusion\ term} \tag{12}$$

$$c_v = \underbrace{\frac{gh^2}{K_0}\frac{\partial(H+h')}{\partial y}}_{surface\ slope} \tag{13}$$

Equation (14) is obtained by integrating (10) with respect to ξ

$$k(\xi)\frac{\partial v}{\partial \xi} = a_v\int_{\xi_0}^{\xi}\Phi(\xi_0,\xi)^2 d\xi' + b_v\int_{\xi_0}^{\xi}k(\xi')\Phi(\xi_0,\xi)d\xi' + c_v\xi + c_1 \tag{14}$$

here c_1 is the constant of integration. The shear strength at the water surface is 0, and when $\xi = 1$ then$\partial v/\partial\xi = 0$ This determines c_v , and Equation (14) is transformed into (15).

$$\frac{\partial v}{\partial \xi} = -\frac{1}{k(\xi)}\left[a_v\int_{\xi}^{1}\Phi(\xi_0,\xi)^2 d\xi' + b_v\int_{\xi}^{1}k(\xi')\Phi(\xi_0,\xi)d\xi' + c_v(1-\xi)\right] \tag{15}$$

Further, the flow velocity distribution in the water-depth direction is obtained by integrating Equation (15) with respect to ξ with the boundary condition of $v = 0$ at $\xi = \xi_0$.

$$v = -\left[a_v\int_{\xi_0}^{\xi}\frac{1}{k(\xi')}\int_{\xi'}^{1}\Phi(\xi_0,\xi)^2 d\xi'' d\xi' + b_v\int_{\xi_0}^{\xi}\frac{1}{k(\xi')}\int_{\xi'}^{1}k(\xi'')\Phi(\xi_0,\xi)d\xi'' d\xi' + c_v\int_{\xi_0}^{\xi}\frac{1-\xi'}{k(\xi')}d\xi'\right] \tag{16}$$

Equation (16) is the flow velocity distribution in the vertical direction obtained from this model. Here, a_v and b_v in (16) are determined by substituting the U and V, obtained from the two-dimensional model, into Equations (11) and (12). Equation (16) has the unknown c_v and it maybe determined as follows: Equation (17) is obtained by considering that the average water depth flow velocity V is given by averaging v at all water depths ($V = \int_{\xi_0}^{1} v d\xi$).

$$V = -\left[a_v\int_{\xi_0}^{1}\int_{\xi_0}^{\xi'}\frac{1}{k(\xi)}\int_{\xi''}^{1}\Phi(\xi_0,\xi)^2 d\xi''' d\xi'' d\xi' + b_v\int_{\xi_0}^{1}\int_{\xi_0}^{\xi'}\frac{1}{k(\xi)}\int_{\xi''}^{1}k(\xi''')\Phi(\xi_0,\xi)d\xi''' d\xi'' d\xi' + c_v\int_{\xi_0}^{1}\int_{\xi_0}^{\xi'}\frac{1-\xi''}{k(\xi'')}d\xi'' d\xi'\right] \tag{17}$$

rearranging (17), gives c_v as

$$c_v = \frac{-V + a_v\int_{\xi_0}^{1}\int_{\xi_0}^{\xi'}\frac{1}{k(\xi'')}\int_{\xi''}^{1}\Phi(\xi_0,\xi)^2 d\xi''' d\xi'' d\xi' + b_v\int_{\xi_0}^{1}\int_{\xi_0}^{\xi'}\frac{1}{k(\xi'')}\int_{\xi''}^{1}k(\xi''')\Phi(\xi_0,\xi)d\xi''' d\xi'' d\xi'}{\int_{\xi_0}^{1}\int_{\xi_0}^{\xi'}\frac{1-\xi''}{k(\xi'')}d\xi'' d\xi'} \tag{18}$$

Substituting c_v into (16), the flow velocity distribution in the water-depth direction is obtained. The vertical flow velocity distribution,W(z) in the water-depth direction can be determined in the same manner.

This method is based on the average water-depth flow velocities (U, V) obtained from the two-dimensional shallow water flow and a logarithmic-parabolic velocity distribution is assumed as a first order approximation of the flow velocity distributions. Individual flow velocities (u,v) are obtained by substituting U and V into the equations of motion in the three-dimensional transverse and downward directions, and secondary flow velocities are estimated by calculating the velocities beyond the logarithmic and parabolic assumption.

The above calculations may seem complicated, however, when $a_v b_v$ andc_v have been calculated once, they may be used as constants, and $\Phi(\xi_0, \xi)$ is also obtained by assuming the logarithmic-parabolic changes (Equation (8)). When the effects of secondary flows have been estimated, accurate flow velocities near the river bed can be calculated very simply.

Comparisons with Experimental Values and Logarithmic Results

Calculated and experimental results were compared to determine the effectiveness of the model. To compare with the two-dimensional model, results with the distribution in the water-depth direction calculated logarithmically were compared with depth averaged water-depth flow velocities obtained by the two-dimensional shallow water flow model.

$$u = U\Phi'(\xi_0, \xi), \qquad v = V\Phi'(\xi_0, \xi) \tag{19}$$

here $\Phi'(\xi_0, \xi)$ is expressed by $(\ln \xi/\xi_0)/(\ln(1/\xi_0) - 1)$ for the logarithm.

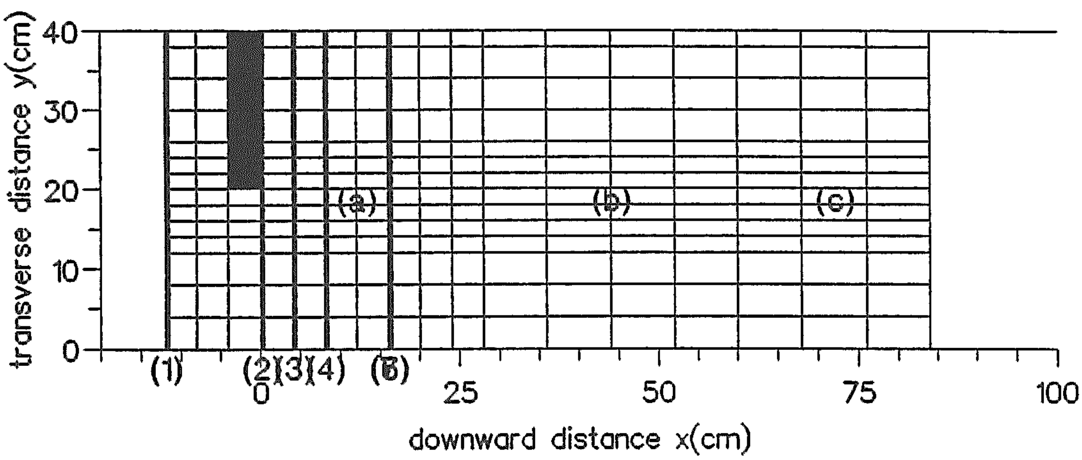

figure-2 measurements conditions

flume length	700cm
flume width	40cm
water level at end	7cm
experimental flume length	100cm
experimental river bed slope	1/1000
spur-dike length	20cm
spur-dike width	4cm
experimental flow	$1.87\ell/s$

table-1 experimental condition

An outline of the measurements and the experimental conditions are shown in Figure-2 and Table-1 (See References Muneta.(1992) for details of the experimental method and conditions). Figure-3 and Figure-4 show the measured (dots) vertical and transverse flow velocities, u and v, and the values calculated (lines) logarithmically and with the modal. Each figure represents a cross section, and the spur-dike projects from the left bank. Downwards in Figure-3 (u) and left to right Figure-4 (v) are positive.

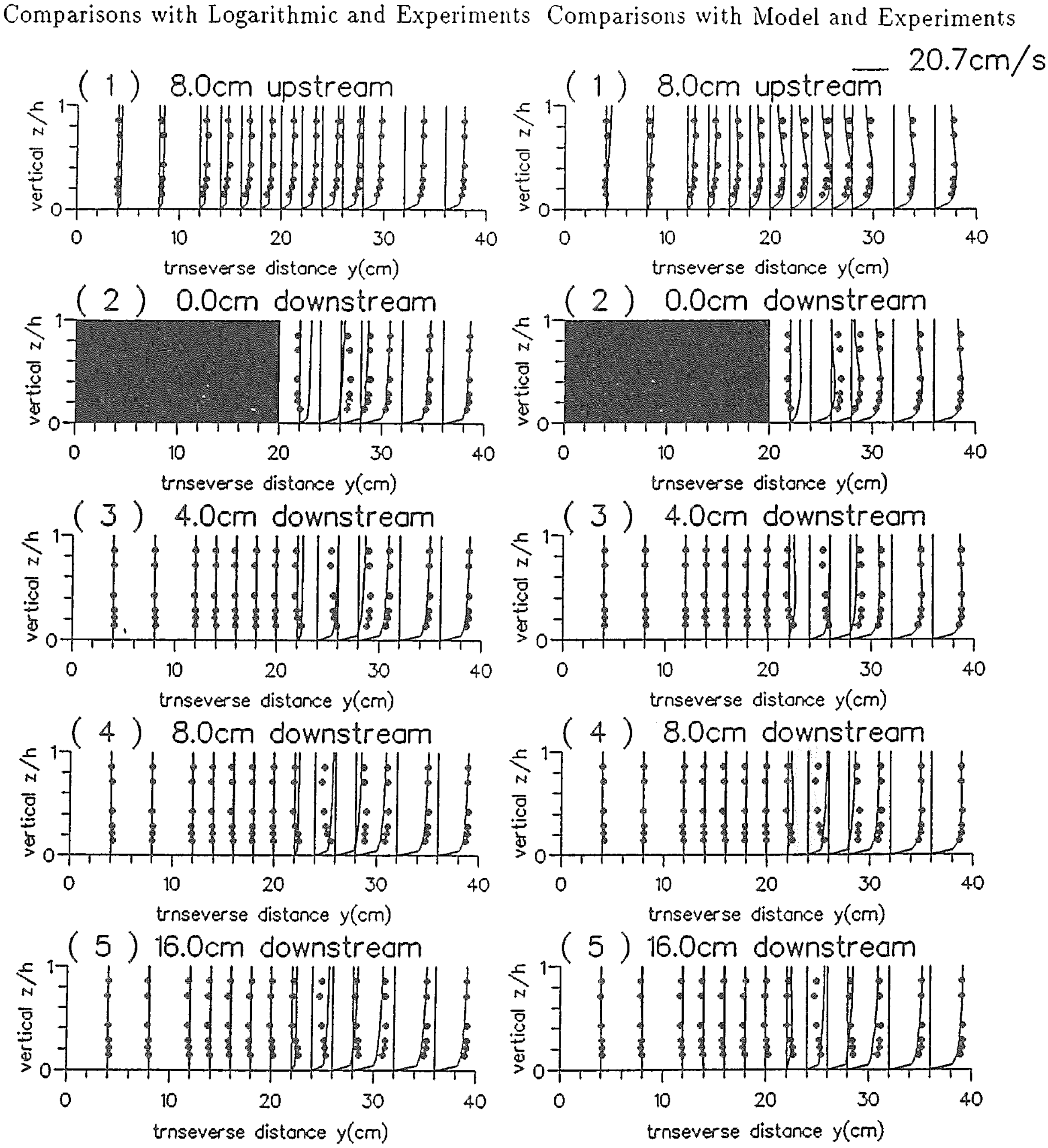

figure-3 the downward flow velocity

First Figure-3, the downward flow velocity u is investigated. In cross section (1), 12cm upstream of spur-dike, the calculated values near the right bank flume wall (opposite to the spur-dike) are somewhat larger than those measured near the river bed, and somewhat smaller near the water surface. The logarithmic distribution gives slightly more accurate results. and the higher flow velocities near the river bed in the acceleration area upstream of the spur-dike are well represented. The measured values for the left bank show that the flow direction near the water surface is the reverse of that near the river bed. This may be because the spur-dike causes a rapid decrease in flow also well reproduced by this model. In (2), the cross section 0cm downstream of the spur-dike, the model shows better reproducibility than the logarithmic rule at all depths. In (3), 4cm downstream of the spur-dike. the same is the case, 18cm and 20cm from the left bank (behind the spur-dike) the characteristics in the separation areas near the surface are opposite to those near the river bed, and well represented. Cross sections (4) and (5). 8cm and 16cm downstream of the spur-dike. also show very accurate calculated results near the river bed.

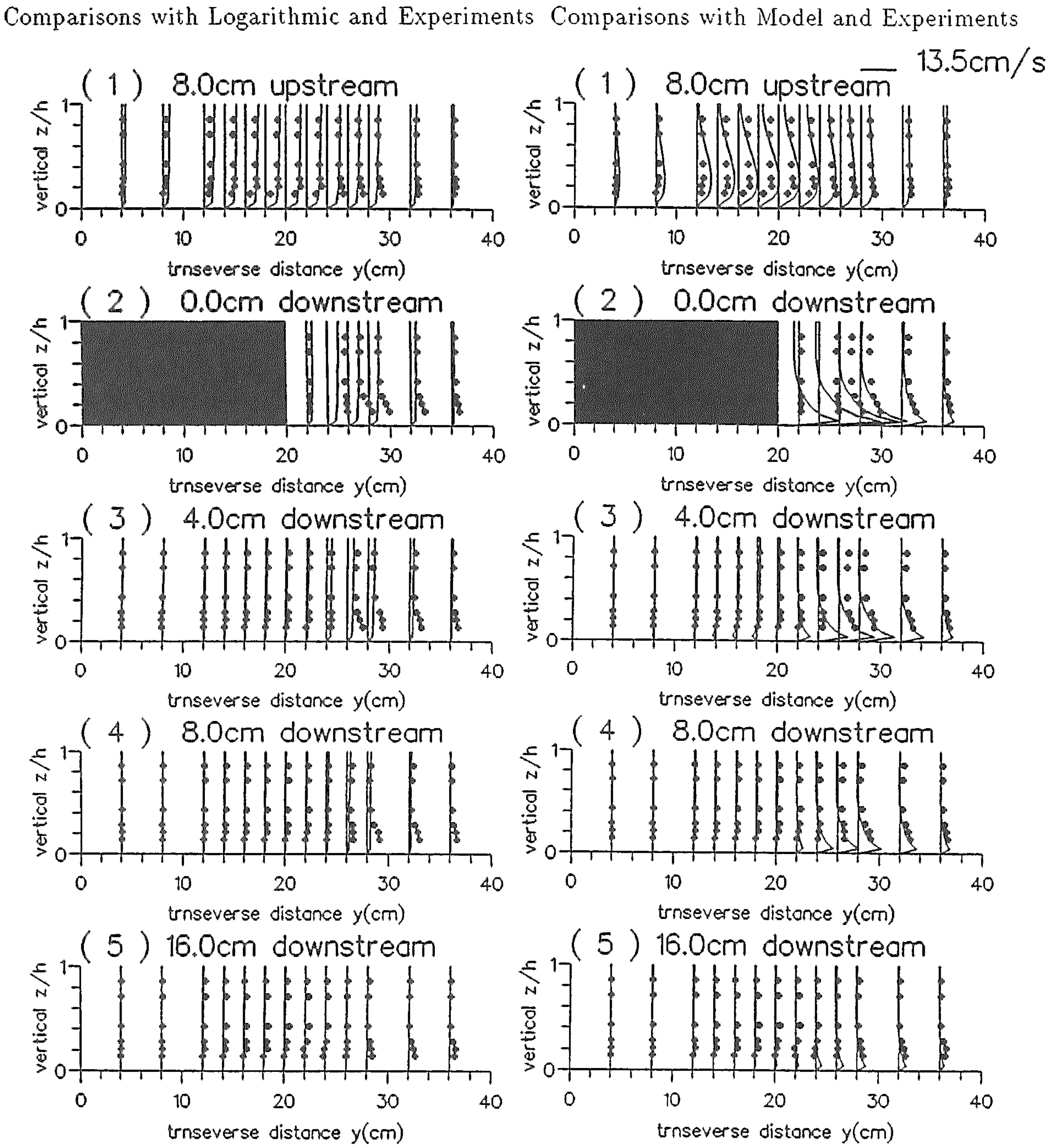

figure-4 the transverse flow velocity

In figure-4 the transverse flow velocity v is investigated. In cross section (1), 12cm upstream of the spur-dike, secondary flows are caused by the spur-dike, and added to the depth averaged flow, resulting in a flow velocity distribution with extraordinary swellings near the river bed. The preset model represents this well while the logarithmic model does not. The v in cross sections (2)-(5) downstream of the end of spur-dikes show secondary flows in both the right and left directions at the horizontal face behind the spur-dike. This may be due to flows accelerating separately to the left (behind the spur-dike) and right banks (the opposite side of spur-dike). The flows are small, but the transverse accelerations may be sufficient to cause secondary flows. Near the river bed, flows into the separation area behind the spur-dike, and flows outside and away from this area may be large. This model reproduces the characteristics of secondary flows well. However, in (2), at the end of the spur-dike, reproducibility is not very good. Shimizu(1991). described the accuracy of the two-dimensional model. "In calculations including transverse structures like spur-dikes, local reproducibility is poor, especially around structures." This model also uses average flow velocities

U and V determined by the two-dimensional shallow water flow model, and the accuracy around spur-dikes may be poor.

Afterword

Integration of this model with the two-dimensional model enabled very good estimates of flow characteristics around structures, especially those due to secondary flows near river beds, without three-dimensional numerical analysis. Flows characteristics near river beds are very important to calculate sediment Flux, and application to calculations of river bed variations can be expected. Particularly, as the application of the three-dimensional model is very difficult for variations in varying river beds, in terms of arrangement of calculation grids in the vertical direction. This model is basically two-dimensional and determines semi-analytically distribution forms in the verticaldirection, and so it may be expected to be very useful in calculating river bed variations. Here the model did not verify secondary flow phenomena in curved river channels, but such estimates may be adequately accurate.

Horizontal mixing due to turbulent diffusion causes flows behind spur-dikes and sediment transport. The vertical mixing detailed in this study has been independently evaluated and applied to numerical analysis. It will be necessary to establish the relation between these phenomena to determine flow structures.

Acknowledgment

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