International Conference on
Environmentally Sound Water Resources Utilization
Bangkok, Thailand, 8-11 November 1993

NUMERICAL CALCULATIONS OF BED DEFORMATION IN THE RANGE OF RESONANT WAVE NUMBERS OF MEANDERING CHANNELS

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ABSTRACT

This paper presents a new method to predict channel bed topography in meandering channels in the range of resonance of free and forced bars. Previous models have not successfully predicted finite bar topography. A depth-integrated version of the three dimensional Reynolds equation is used to calculate the mutual effect of bed and channel geometry. The intensity of secondary flow is calculated from the first order differential equation for momentum in the transverse direction. A new technique is introduced to account for the effect of changes in channel width caused by sediment deposition in the inner bend. Calculated results are successfully compared with a series of experiments conducted in the resonant wave number range.

INTRODUCTION

Prediction of maximum scour depth in alluvial rivers is important for the design of river protection works. Free bars, like alternate bars and forced bars in bends, are very important factors to determine scour depths of river beds. There have been a number of experimental and theoretical studies of the two phenomena. Investigation of the interaction between free and forced bars, was pioneered by Kinoshita and Miwa(1974). Hasegawa and Yamaoka(1980) applied a quantitative treatment. This was followed by theoretical methods where both free and forced bars develop resonantly at a given frequency of meandering channels [Blondeaux and Seminara(1985) and by Parker and Johannessen(1989)].

Shimizu and Itakura(1990) and Watanabe *et al.*(1990) have presented a simple numerical model and equation using the linear sums of wave height of free and forced bars in order to develop a practical method for predicting the heght of bars in rivers.

This paper reexamines studies of interactions between free and forced bars, and will first show that the simple numerical calculation model and the equation based on linear theory does not give very accurate results when the meander frequency is in the resonance range.

Next, the paper will present a more widely applicable calculation method by improving the numerical model and comparing the results with those of movable bed experiments conducted by Colombini *et* *al.*(1990). Finally, the interaction between free and forced bars is studied over a wide range of conditions, including the resonance domain, by performing numerical experiments under the condition that the meander frequency is remarkably changed with a given curvature radius using the proposed numerical calculation method.

PROBLEMS WITH EXISTING METHODS

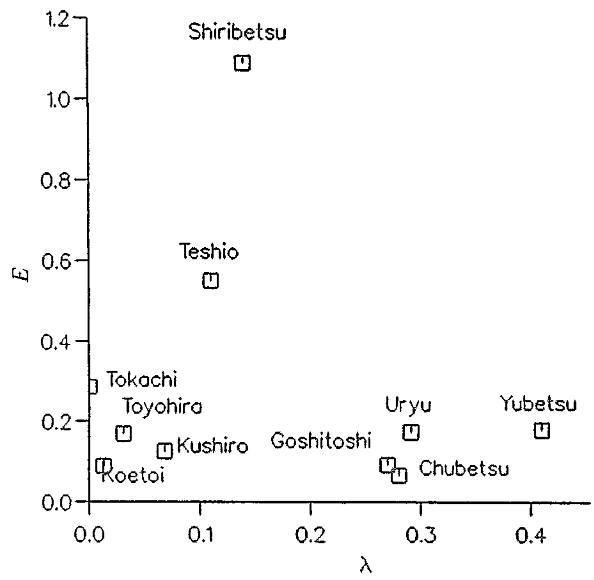
Assuming that the bar height in meandering rivers is the linear sum of the bar height due to free (alternate) and forced bars, Watanabe *et al.*(1990) presented a simple equation and applied it to 13 rivers in Hokkaido, Japan. Figure–1 shows the relation between Eqs. 1 and 2. The former plots the average of the squares of errors of the equation proposed by Watanabe *et al.*(1990) against the measured values of E, and the latter shows the meandering wavenumber λ in each river. E and λ are defined as follows.

$$E = \frac{1}{m}\sum_{m}(\eta_{cal} - \eta_{obs})^2 \tag{1}$$

$$\lambda = \frac{2\pi}{\tilde{L}}\tilde{B} \tag{2}$$

Here η_{cal} and η_{obs} are calculated and measured values of the nondimensional maximum scour depth below the averaged river bed height scaled by the mean water depth; m is the number of sectional samples in each river; $\tilde{B}$ is 1/2 of the mean river width; $\tilde{L}$ is the mean meander wavelength; and $\tilde{B}$ and $\tilde{L}$ are visually estimated from the river plane figure. Figure–1 shows that errors are large around the meander wavenumber $\lambda = 0.2$, but are otherwise within an acceptable range. Blondeaux and Seminara(1985) reported that flows and river bed forms developed remarkably in the $\lambda = 0.1$–0.2 range. In this range, wave characteristics of free bars and the meander plane were resonant.

The method of Watanabe *et al.*(1990) is based on the linear sum of bar heights due to free and forced bars. Thus, it cannot estimate the bar height correctly in a resonant range, where nonlinear effects come into play.



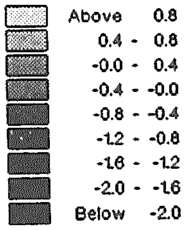
Figure–1 Result of scour depth prediction for 13 rivers in Hokkaido, Japan. λ is the dimensionless meander wave number and E is the mean square difference in maximum scour depth between observations and predictions using Watanabe's (1990) equation.

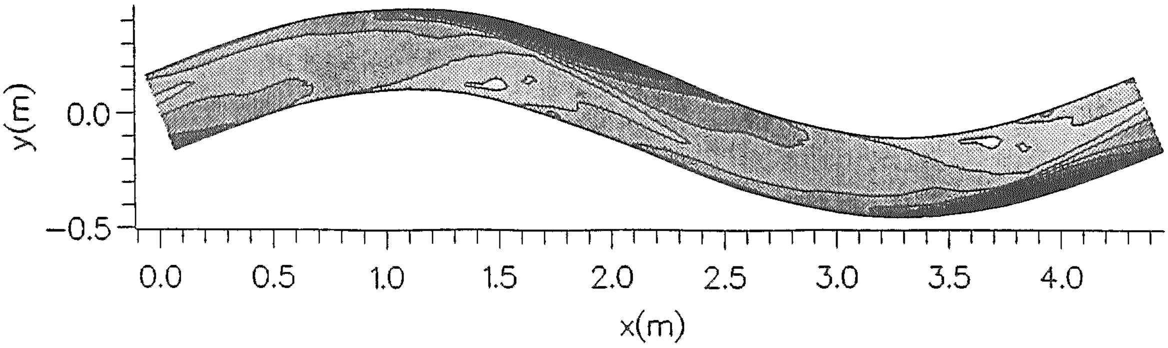
Table–1 Experimental Condition of W25Q14

Discharge (ℓ/sec)	$\tilde{Q}$	1.39
$\frac{1}{2}$ Channel Width (m)	$\tilde{B}$	0.175
Mean Depth (cm)	$\widetilde{D_0}$	1.40
Meander Wave Length (m)	$\tilde{L}$	4.40
Meander Amplitude (m)	$\tilde{a}$	0.28
Dimensionless Wave Number	$\lambda = 2\pi\tilde{B}/\tilde{L}$	0.25
Dimensionless Wave Amplitude	$a = \tilde{a}/\tilde{B}$	1.6
Dimensionless Radius	$\nu = a\lambda^2/2$	0.05
Dimensionless Channel Width	$\beta = \tilde{B}/\widetilde{D_0}$	12.50
Dimensionless Grain Diameter	$d_s = \tilde{d}_s/\widetilde{D_0}$	0.054
Dimensionless Shear Stress	θ	0.067
Froude Number	Fr	0.77
Time of Experiment (minute)		240

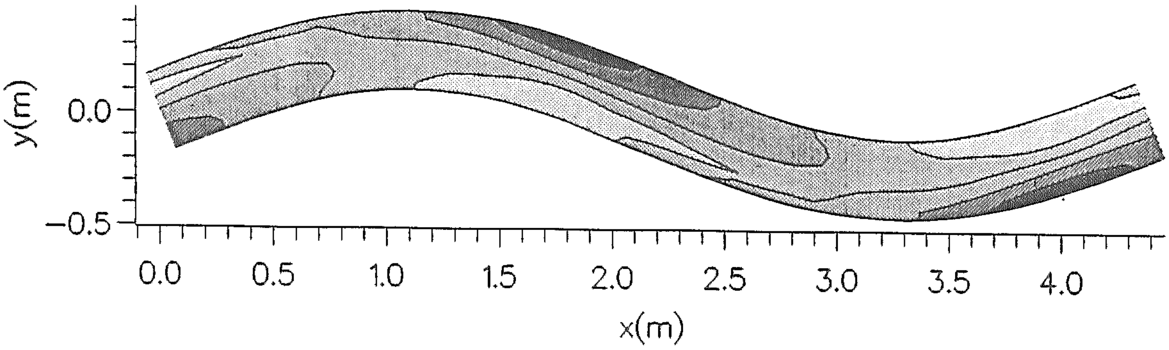
To examine predictions of bed deformation with and without resonance, the calculation method of two-dimensional flow and bed deformation proposed by Shimizu and Itakura(1990) was used. Calculations using this numerical model are compared with movable bed meandering channel experiments (W25Q14) performed by Colombini *et al.*(1990) as shown in Table–1.

Figures–2 and –3 show the contour lines of channel bed for the experimental and calculated results of W25Q14 (Method 1). The contour lines are represented by the deviation from the initial channel bed of the experiment scaled by the mean water depth. The method of calculation for Figure-3, is neary identical to that proposed by Shimizu and Itakura(1990), and thus it was possible to calculate successive decreases in water surface width by eliminating the computational grids when bars emerged above the water surface (water depth=0). Figure–3 shows that the locations of scour and deposition are generally well reproduced, however the amounts of scour are underestimated.





Figure–2 Experimental result of bed configuration of W25Q14



Figure–3 W25Q14 Calculated bed configuration with existing two-dimensional model by Shimizu and Itakura(1990) (Method 1)

It is impossible to predict the wave heights of bars (scour depth) accurately with the linear model and the simple two–dimensional model where the meandering wavenumber is close to the resonance domain. Because it is very inconvenient in river planning works when scour depths are underestimated (as in Figure–3), there is need to improve the method for engineering purposes.

TWO-DIMENSIONAL MODEL CONSIDERING SECONDARY FLOW

The calculation described in the previous section has some problems. First, the influence of the vertical velocity distribution is ignored in the flow calculation, and it is impossible to accurately describe the development and damping of the secondary flow. The secondary flow is important to describe the bed shear stress and sediment transport rate vector. Although this may be solved by using a three-dimensional model, such an approach is problematic, where the water depth at the bar is calculated to be zero, due to the arrangement of the computational grid in the vertical direction. Therefore, this paper will present a method in which the vertical distribution of velocity is integrated and included in two the dimensional model.

Flow Equations

The following dimensionless values and equations are used. The dimensional values are indicated with $\widetilde{\ }$, and the reference values for the basic flow with a subscript 0.

$$\tilde{u} = \widetilde{U_0}u \quad \beta = \frac{\tilde{B}}{\widetilde{D_0}} \quad \zeta = 1 + \frac{(z - F_0^2 H)}{D}$$

$$\tilde{v} = \widetilde{U_0}v \quad \tilde{H} = \widetilde{D_0}{F_0}^2 H \quad U = \int_{\zeta_0}^{1} ud\zeta$$

$$\tilde{s} = \tilde{B}s \quad \tilde{\nu_t} = \sqrt{C_0}\widetilde{U_0}\widetilde{D_0}\nu_t \quad V = \int_{\zeta_0}^{1} vd\zeta$$

$$\tilde{n} = \tilde{B}n \quad N = \frac{\tilde{R}}{\tilde{R}+\tilde{n}} = \frac{R}{R+\nu n} \quad u = u_0(\zeta)U(s,n)$$

$$\tilde{z} = \widetilde{D_0}z \quad \tilde{R} = \widetilde{R_0}R \quad \widetilde{Q_s} = \sqrt{\left(\frac{\rho_s}{\rho}-1\right)g\tilde{d_s}^3}\,Q_s \quad v = \nu\Gamma(s,\zeta)U(s,n) + u_0(\zeta)V(s,n)$$

$$\tilde{t} = \frac{\tilde{B}}{\widetilde{U_0}}t \quad \widetilde{Q_n} = \sqrt{\left(\frac{\rho_s}{\rho}-1\right)g\tilde{d_s}^3}\,Q_n \quad \tau = \sqrt{{\tau_s}^2 + {\tau_n}^2}$$

$$\nu = \frac{\tilde{B}}{\widetilde{R_0}} \quad \tilde{\eta} = \widetilde{D_0}\eta \quad Q_0 = \frac{\sqrt{\left(\frac{\rho_s}{\rho}-1\right)g\tilde{d_s}^3}}{(1-p)\widetilde{D_0}\widetilde{U_0}} \quad \theta = \frac{\widetilde{U_0}^2}{\left(\frac{\rho_s}{\rho}-1\right)gd_s}\tau$$

Here s, n, and z are coordinate axes in the flow, transverse, and vertical directions; u and v are the velocity components in s and n directions, R is the radius of curvature along the the channel center line scaled by $\widetilde{R_0}$, the center line radius of curvatur at the bend apex; H is the water surface elevation; D is the depth; ν_t is the momentum diffusion coefficient; η is the bed elevation; t is time; Q_s and Q_n are the bed load transport rate per unit width in the s and n directions; ρ and ρ_s are the densities of the water and the bed material; g is the acceleration of gravity; d_s is the grain size of the bed material; p is the void ratio of the bed material; U and V are the depth averaged value of u and v; τ_s and τ_n are the channel bed shear in the s and n directions; and θ is the dimensionless bed shear stress.

When the three-dimensional Reynolds Equation and the continuity equation are integrated in the vertical direction and expressed dimensionlessly using the above elements, the following equations are obtained:

$$N\frac{\partial}{\partial s}\left[DU^2\right] + \frac{\partial}{\partial n}\left[D(\nu U^2 k_2 + UV)\right] + \frac{2\nu ND}{R}(\nu U^2 k_2 + UV) = -ND\frac{\partial H}{\partial s} - \beta\tau_s \tag{3}$$

$$N\frac{\partial}{\partial s}\left[D(\nu U^2 k_2 + UV)\right] + \frac{\partial}{\partial n}\left[D(\nu^2 U^2 k_1 + 2\nu UVk_2 + V^2)\right]$$
$$-\frac{\nu ND}{R}(U^2 - \nu^2 U^2 k_1 - 2\nu UVk_2 - V^2) = -D\frac{\partial H}{\partial n} - \beta\tau_n \tag{4}$$

$$N\frac{\partial(UD)}{\partial s} + \frac{\nu NVD}{R} + \frac{\partial(VD)}{\partial n} = 0 \tag{5}$$

in which k_1, k_2, and k_3 are the coefficients which express the interference between the main and the secondary flow [Seminara and Tubino(1989)]. To determine these coefficients, it is necessary to predict the intensity of the secondary flow. Referring to Ikeda and Nishimura(1986), and Johannessen and Parker(1989), the following transport equation for the secondary flow is introduced along the center of the channel, by solving this equation numerically, the intensity of the secondary flow can be determined.

$$u_0\frac{\partial\Gamma}{\partial s} - \frac{{u_0}^2}{R} = -\frac{\partial H_1}{\partial n} + \beta\sqrt{C_0}\frac{\partial}{\partial\zeta}\left(\nu_t\frac{\partial\Gamma}{\partial\zeta}\right) \tag{6}$$

Here Γ and H_1 are the coefficients of the first order to ν by which v and H are expanded purturbatively ($v = 0 + \nu\Gamma, H = H_0 + \nu H_1$); u_0 is a vertical logarithmic velocity distribution; $\partial H_1/\partial n$ is determined by the conditions of $\int_{\zeta_0}^{1}\Gamma d\zeta = 0$ and $\Gamma(\zeta = \zeta_0) = 0$. From the distribution of Γ, k_1, k_2, and k_3 are expressed by the following equations.

$$k_1 = \int_{\zeta_0}^{1} \Gamma^2 d\zeta, \qquad k_2 = \int_{\zeta_0}^{1} u_0 \Gamma d\zeta, \qquad k_3 = \left[\frac{\frac{\partial \Gamma}{\partial \zeta}}{\frac{\partial u_0}{\partial \zeta}}\right]_{\zeta=\zeta_0} \tag{7}$$

The bed shear stress along the channel bed, τ_s and τ_n, are expressed using the depth-averaged velocity:

$$\tau_s = CU\sqrt{U^2+V^2}, \qquad \tau_n = C(V+\nu U k_3)\sqrt{U^2+V^2} \tag{8}$$

C is the bed friction coefficient, which is expressed by the equation by Engelund and Hansen(1967) for flat bed:

$$C^{-1/2} = 6 + 2.5\ln\left(\frac{D}{2.5d_s}\right) \tag{9}$$

Sediment Transport Equations

The bed load transport rate in the direction of the bed shear stress vector, ϕ, is calculated by the Meyer-Peter-Müller(1948) equation:

$$\phi = 8(\theta - \theta_c)^{1.5} \tag{10}$$

Where θ_c is the dimensionless critical shear stress. The bed load transport rate in the bed shear stress direction, $\hat{s}$, ans $\hat{n}$, the direction tangential to $\hat{s}$, are expressed by:

$$\phi = \sqrt{Q_{\hat{s}}^2 + Q_{\hat{n}}^2}, \qquad \frac{Q_{\hat{n}}}{Q_{\hat{s}}} = -\frac{r}{\sqrt{\theta\beta}}\frac{\partial \eta}{\partial \hat{n}} = -\frac{r}{\sqrt{\theta\beta}}\left[\frac{\partial \eta}{\partial n}\frac{\tau_s}{\tau} - \frac{\partial \eta}{\partial s}\frac{\tau_n}{\tau}\right] \tag{11}$$

in which $r = \sqrt{\theta_c/(\mu_s\mu_k)}$ (μ_s and μ_k are the coefficients of static and dynamic friction), given by Hasegawa's(1984) equation, and $\tau = \sqrt{\tau_s^2 + \tau_n^2}$. The sediment transport rate in the $\hat{s}$ and $\hat{n}$ directions are calculated by Eq. 11, and then components in the s and n directions are derived by:

$$Q_s = Q_{\hat{s}}\frac{\tau_s}{\tau} - Q_{\hat{n}}\frac{\tau_n}{\tau}, \qquad Q_n = Q_{\hat{s}}\frac{\tau_s}{\tau} + Q_{\hat{n}}\frac{\tau_n}{\tau} \tag{12}$$

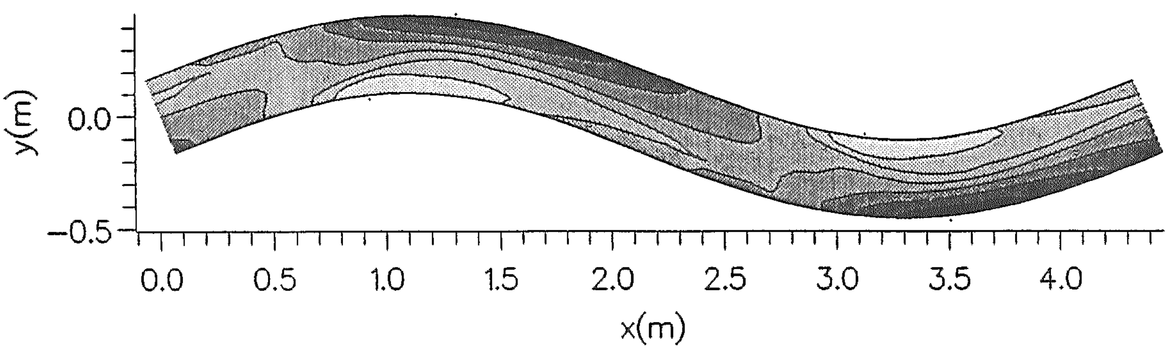
The time-dependent change of channel bed elevation is determined by the continuity equation for sediment transport.

$$\frac{\partial \eta}{\partial t} + Q_0\left(N\frac{\partial Q_s}{\partial s} + \frac{\nu N}{R}Q_n + \frac{\partial Q_n}{\partial n}\right) = 0 \tag{13}$$

Example of Application

The above equations are applied to the conditions of the movable bed experiment W25Q14 in the previous section. The calculation is performed using the finite difference method.

Starting from the initial bed (flat), the bed elevation after 240 minutes is calculated as in the experiment. Figure–4 shows the calculated bed configuration (Method 2). Compared to Figure–3, the calculated results are more accurate, however, the maximum scour depth is still underestimated.



Figure–4 W25Q14 Calculated bed configuration considering interraction between the secondary and main flow (Method 2)

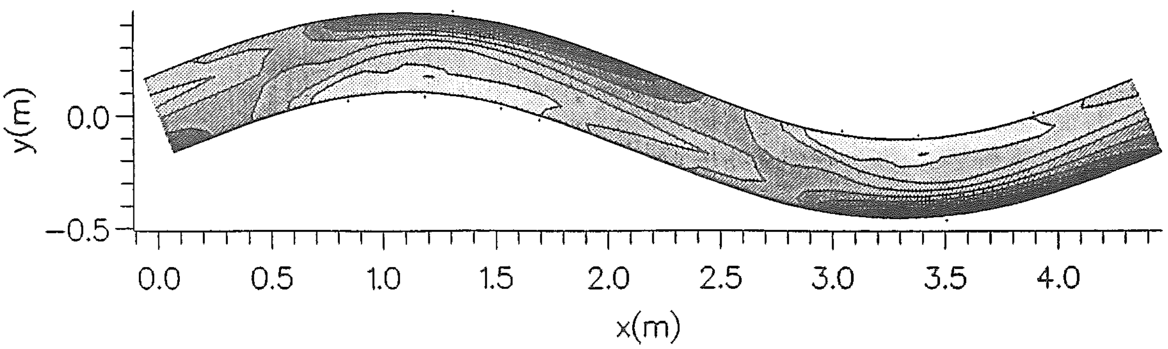
Improved Method for Secondary Flow Intensity

The equation to calculate the intensity of the secondary flow in Method 2 described above is identical to that of Seminara and Tubino(1989), in which Γ is obtained from Eq. 6 under the assumption of quasi-uniform flow. However, Γ is originally considered to change with the change in flow conditions due to the deformation of the bed. Here, we assume that Γ is expressed by the following equation by considering that the intensity of the secondary flow is proportional to the local flow depth, as suggested by Ikeda(1974).

$$\Gamma(s,n,z) = \Gamma_0(s,z)\frac{D(s,n)}{D_0} \tag{14}$$

where Γ_0 is the Γ of the initial bed topography obtained by Eq. 6.

Figure–5 shows the calculated bed configuration in which the secondary flow is calculated by Eq. 14. The calculated scour depth agrees very well with the observed one. This method appears to yield accurate results for bed deformation even in the range of resonance.



Figure–5 W25Q14 Calculated bed configuration considering changes in intensity of secondary flows corresponding to local flow depths (Method 3)

NUMERICAL EXPERIMENT OF INTERFERENCE BETWEEN FREE AND FORCED BARS

To investigate the interaction between free and forced bars, differences in channel geometry were studied with numerical calculations. The meander wave length was varied while the channel width, water depth, and curvature radius were held constant.

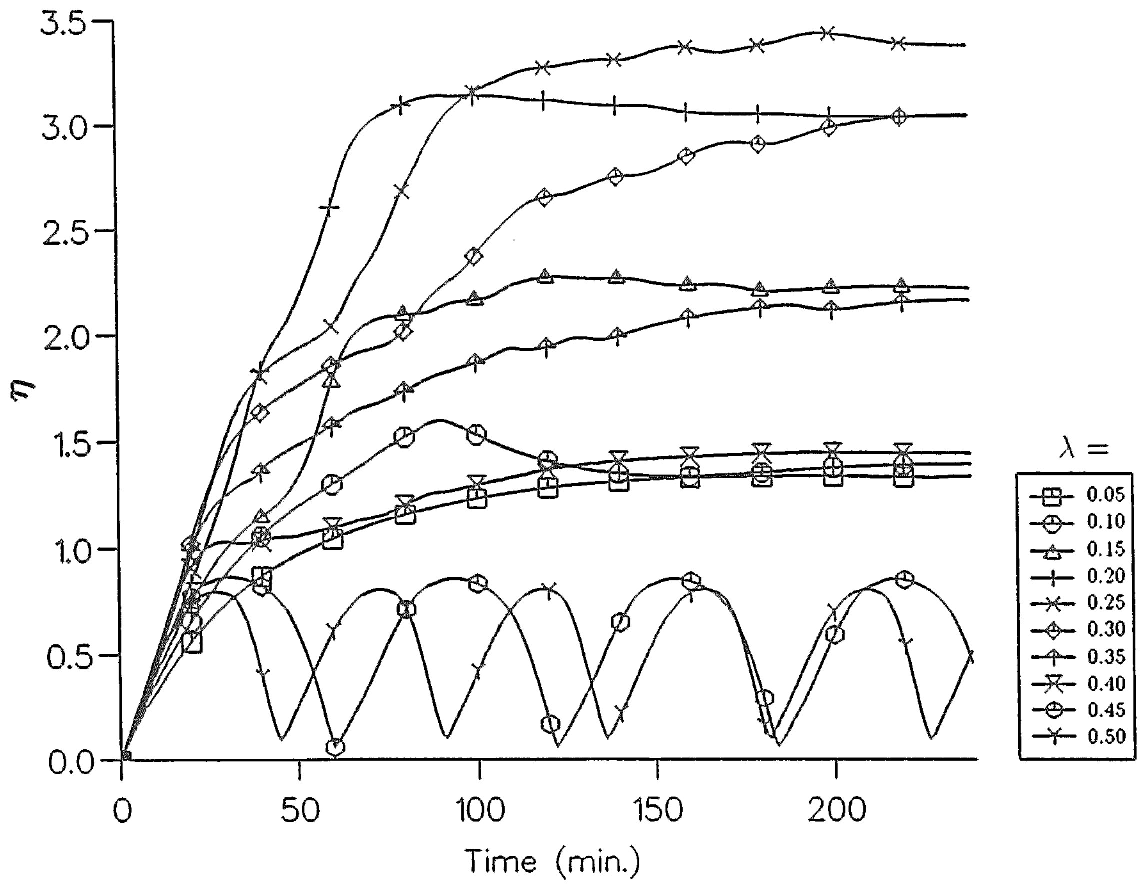
The experimental conditions of Colombini *et al.*(1990) (Table 2) were used and a series of calculation were performed using Method 3. Figure–6 shows the time change of the dimensionless maximum scour depth for various meander wavenumbers calculated by the model. Figure–7 shows the calculated relation between the dimensionless maximum scour depth and the dimensionless meander wavenumber λ, compared with the experimental results of Colombini *et al.*(1990). In Figure–6, the periodic behavior of the maximum scour depths with $\lambda = 0.45$ and 0.50 result from the development and migration of alternate bars. There are no alternate bars in the range between $\lambda = 0.40$–0.10. However, these alternate bars appear again at $\lambda = 0.05$, at which point the channel is nearly straight. Experimental values between $\lambda = 0.15$–0.30 agree well with the calculated ones in Figure–7. As the scour depth decreases smoothly outside the range of experiment, there are no abrupt resonant phenomena in the range of resonance, as presented by Blondeaux and Seminara(1985). The method of Watanabe *et al.*(1990), based on a linear theory, gives $\eta = 0.82$, which agrees with the experimental results when λ is small or large, however it underestimates η in the range of $\lambda = 0.1$–0.3.

SUMMARY

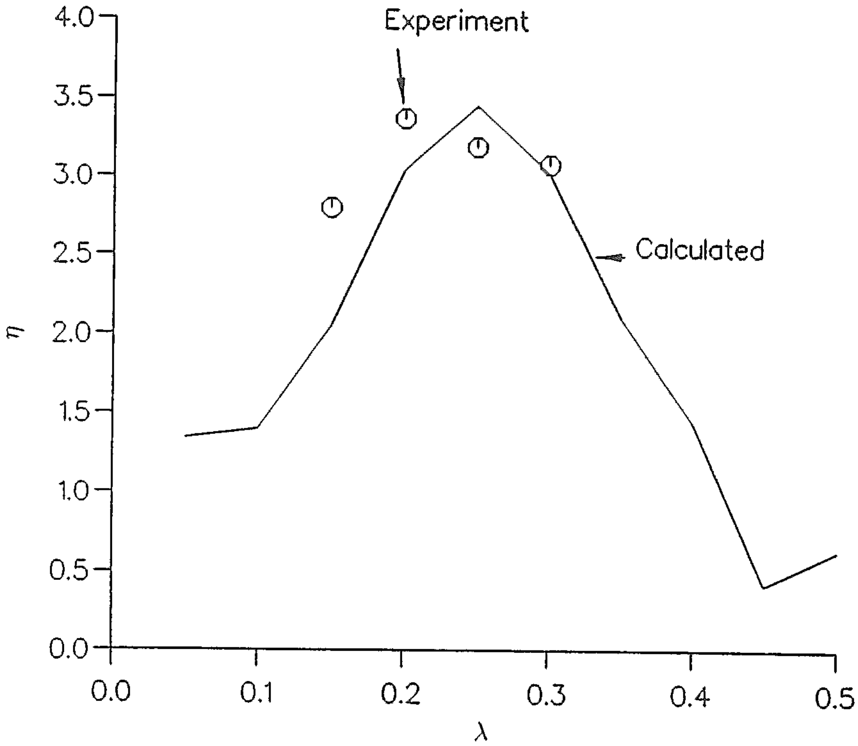
This paper examines existing methods for estimationg bed scour depth in terms of the interraction between free and forced bars. As a result, a two-dimensional model applicable over a wide range of conditions, including the resonance domain, is presented. This model will help to improve the prediction of bed topography in practical applications. One interesting result of this investigation is the possibility that the scour depth changes over a very wide range, from half to four times the water depth, as the meander wavelength is changed holding the radius of curvature, channel width, water depth, and bed materials constant (Figure–7). This result was interpreted in terms of the interaction of free and forced bars.

Table–2 Hydraulic Conditions for Numerical Experiment

$\tilde{Q}$ (ℓ/s)	1.27
$\tilde{B}$ (m)	0.175
$\tilde{D}_0$ (cm)	1.25
ν	0.05
β	12.50
d_s	0.061
θ	0.06
Fr	0.83
λ	0.05 ~ 0.5



Figure–6 Change in dimensionless maximum scour depth η over time.



Figure–7 Relation between dimensionless maximum scour depth η and dimensionless meander wavenumber λ

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