A STUDY OF RUNOFF MODELS CONSIDERING GROUND WATER FLOWS

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Abstract

Understanding the physical mechanism of rainfall and runoff in catchment areas is important for the management of catchment areas and rivers. Presently, the concentration-type models generally used in river planning (such as hydrograph analysis by storage functions). These concentation-tye models represent the rainfall and runoff phenomena with adjustable parameters, but do not evaluate their physical significance. A distribution-type runoff model which represents the physical mechanisms is needed to incorporate the varying conditions of catchment areas. Such a model is approximated in this study by a model of saturated and unsaturated percolation flows using the Richards'[1] Equation to represent the rainfall and runoff mechanism of slopes. This model is verified by comparing calculations from it with aset of the rainfall percolation runoff experiments on two-dimensional slopes.

INTRODUCTION

In general, discharge into the rivers by rainfall is divided into surface and underground flow. Actually, some part of the surface flow becomes ground water flow during a flood. Likewise underground flow turns into surface flow as a result of saturation of the soil layer.

This phenomenon complicates the analysis of open channel flows. This study concentrates on the flows of underground water.

In the study of Ichikawa and Yamamoto[2] the mechanisms of the transport of rainfall on slopes was verified using experimental results obtained from a model of underground flow and from calculations. A simple experiment was carried out to measure the transport of rainfall as both surface and underground flows. Experimental and numerical models were used to understand the physical processes of seepage, rainfall and runoff. Discussion and comparison between the measured values and calculalted results are presented. The findings of this study can be used to improve natural runoff models.

EXPERRIMENTAL INSTRUMENTATION

The experimental consisted of a soil layer, a rainfall generation facility, and runoff as shown in Figure-1, as well as a data display and storage facility. The functions, sizes, and characteristics are as follows: The soil layer is placed on a tilting flume, 30cm wide, 500cm long, and 40cm deep, filled with quartz sand (particle size ϕ=0.2mm) in the uppermost 480cm of the slope and gravel in the final 20cm at the downstream end to prevent quartz sand from being washed out. Tensiometers are installed at 21 points along the sidewall for the measurement of pressure. For the rainfall generation, three vinyl chloride pipes are placed along the direction of the slope, with 0.4mm holes at 5cm intervals and hanging cotton threads are used to form raindrops to simulate rainfall. A net to break the fall of the drops is set slightly above the soil layer to prevent the development of cavities on the soil layer and to distribute the water widely. To supply a set rainfall volume, tap water is stored in a water tank, and supplied while maintaining a stable water level in the tank. The rainfall intensity is controlled by a valve attached to each vinyl chloride pipe. A runoff counter to measure runoff per unit time is set at the downstreamend. To determine the accuracy of the runoff counter, a tipping-bucket rain gauge is also placed to measure the runoff discharge.

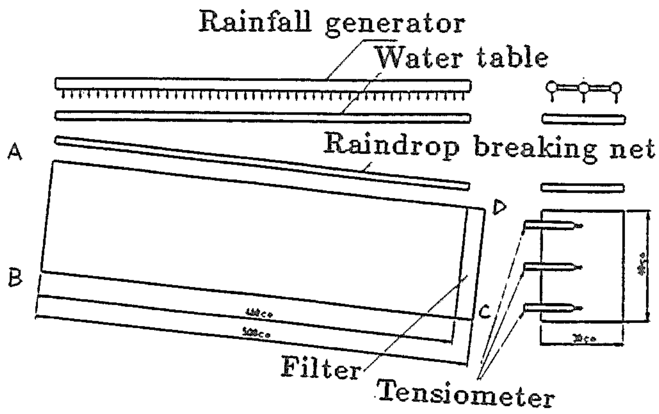

Figure-1 Experimental Installation

The data display and storage reads data from the tensiometers and the runoff counter at set time intervals for predetermined periods, displays it on a personal computer, and all the data is stored into a hard disc.

OUTLINE OF EXPERIMENTS

The experiment was conducted under four conditions with rainfall intensities of 10mm/hr and 30mm/hr on 1/5 and 1/10 slopes. Before the start of the experiment, the data display and storage was reset. The rainfall intensity was controlled in each experiment, averaging three measurements. Data was read at one-minute intervals, and the completion time was based on experiments conducted the previous year. The data obtained from the experiment comprises the negative soil pressure (suction ψ) at 21 measuring points (arrangement shown in Figure-2) and the accumulated runoff. The values are stored into a hard disc, and displayed on a monitor as a volt-time graph. The water was stopped depending on the progress of the experiment, after the runoff had reached a steady state In some Runs, there was surface runoff, and was measured per minute with a

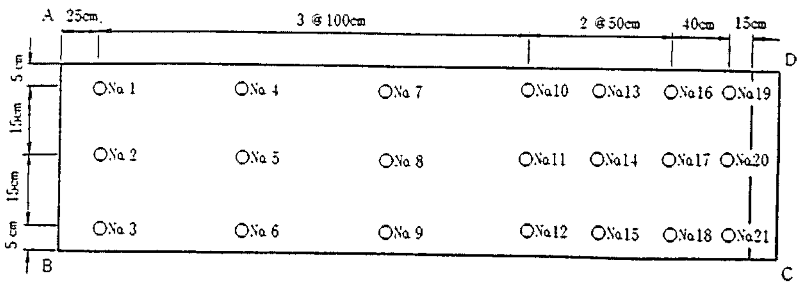

Figure-2 Arrangement of Tensiometer

Table-1 Experimental Conditions

Run-no.	Gradient	Rainfall time (hr)	Rainfall intensity (mm/hr) Start	Stop
1	1/5	6.0	10.72	10.44
2	1/10	8.0	10.90	10.26
3	1/5	4.5	30.85	29.63
4	1/10	3.0	32.05	30.46

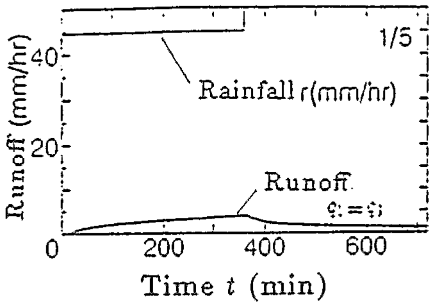

Figure-3 Changes in Rainfall Intensity r and Runoff (Run 1)

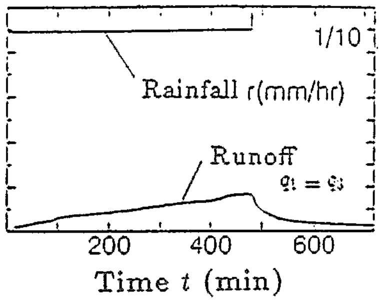

Figure-5 Changes in Rainfall Intensity r and Runoff (Run 2)

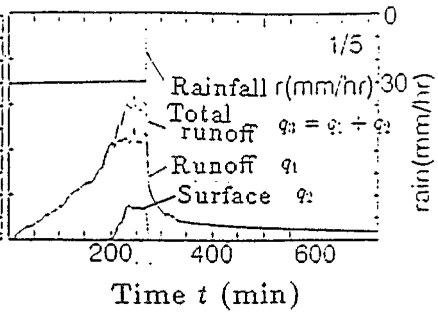

Figure-7 Changes in Rainfall Intensity r and Runoff (Run 3)

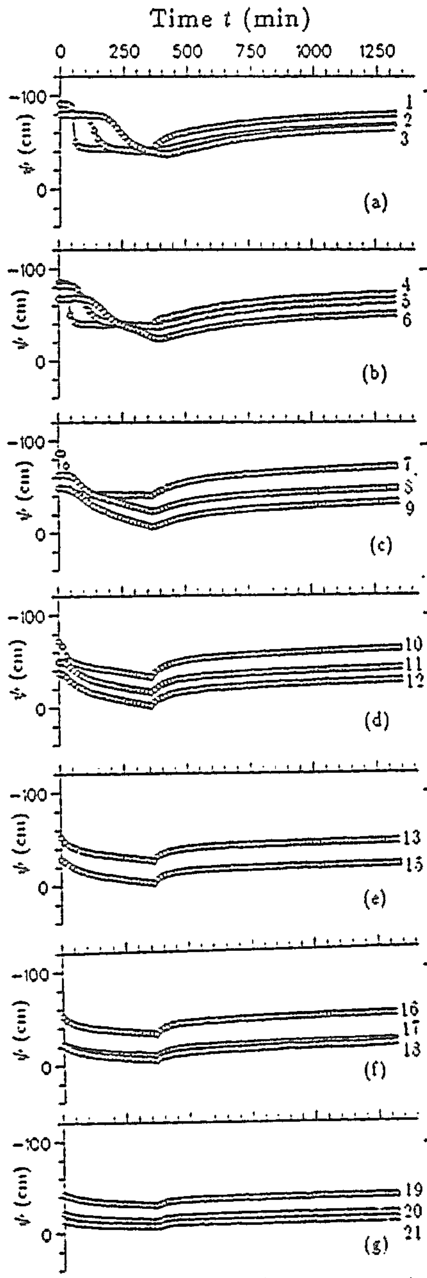

Figure-4 Changes in Suction ψ (Run 1)

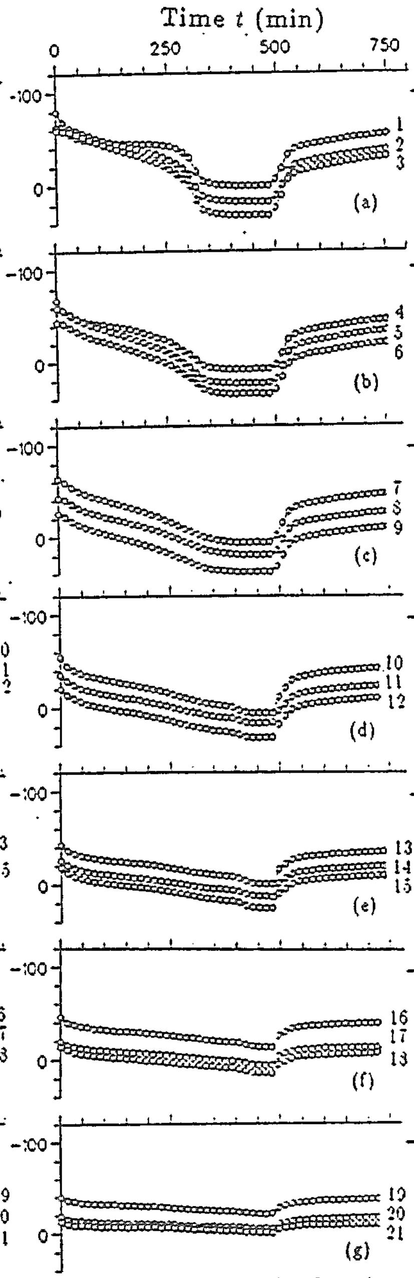

Figure-6 Changes in Suction ψ (Run 2)

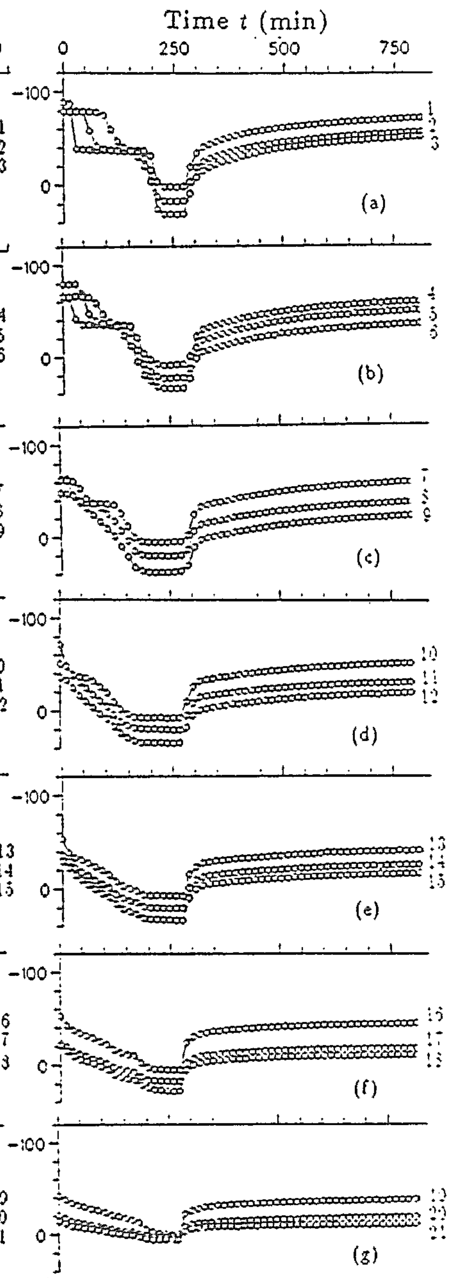

Figure-8 Changes in Suction ψ (Run 3)

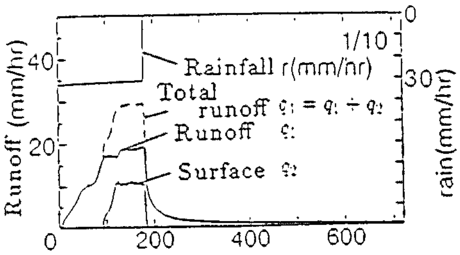

Figure-9 Changes in Rainfall Intensity r and Runoff (Run 4)

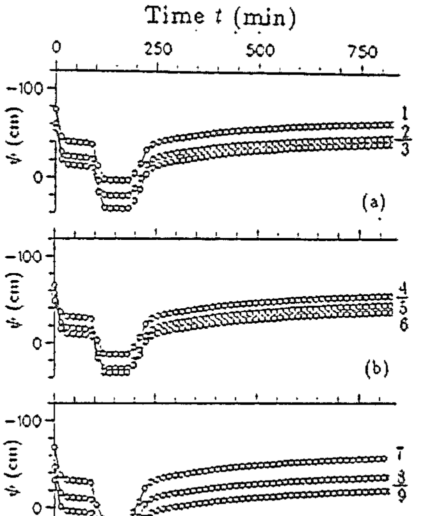

Figure-10 Changes in Suction ψ (Run 4)

measuring cup. After the rainfall had been stopped, measurement of the negative pressure and runoff discharge continued for a set time. Before the start of experiments, the soil was drained to a uniform initial condition.

EXPERIMENTAL RESULTS AND DISCUSSION

Figure-3 to -10 show hydrographs of runoff discharges and changes in suction with time. Figures-3, -5, -7, and -9 represent the infiltration discharge [q_1 (mm/hr)], the surface discharge [q_2 (mm/hr)],

and the total discharge [q_3 (= $q_1 + q_2$)]. Numbers beside the lines in Figures-4, -6, -8, and -10 refer to the positions of the tensiometers shown in Figure-2.

Consideration of the rainfall infiltration process was based on the data obtained from the experiment.

Groundwater flow occurred at the end of flume immediately after the rainfall started. This initial flow was probably caused by water already saturated into the soil layer, and when the rainfall began a flow potential occurred pushing saturated water out of the end of the flume. The suction ψ at each point with tensiometers changed to 0 as runoff discharge increased. In some experimental runs, suction ψ immediately saturated beyond 0 at the lower depth of the flume. Free surface water could be predicted along the line of suction ψ which turned into positive. Experiment 3 with a rainfall intensity of 30 mm/hr and a 1/5 slope was used to consider the movement of free water surface. Data obtained in this Run are presented in Figures-7 and -8. No free surface water existed because not all of the flume was saturated when it began to rain. Free surface water appeared with an increase in rainfall and storage capacity and was saturated in certain places as time passed. The free surface water appeared at a location slightly downstream from the middle of the flume slope. Free surface water was seen to have ascended as suction ψ changed when rainfall continued. In a Run where rainfall intensity was low and soil moisture content reached a certain level, the underground flow appeared steady at some value.

In this experiment, free surface water increased at the top surface of the soil layer. At first, it appeared in the middle of the flume then spread all over the flume. At this time, the water appeared in the section where the free surface water first got to the surface of the soil layer. Then the free water spread and surface flow began. Meanwhile with the diminished capacity of water in the soil, the underground flow became steady and exceeded the surface flow which also eventually became steady. Experiment 4 revealed similar discharge patterns although the ratio of underground flow and surface flow was different due to different slopes. At some slopes the discharge was thought to be the same.

SATURATED AND UNSATURATED MODELS OF SEEPAGE FLOWS

The experimental results were reproduced by calculations the using saturated and unsaturated infiltration theory of Richards[1] .

The continuity equation of soil water content is expressed by:

$$\frac{\partial\theta}{\partial t} = -\Big(\frac{\partial V_x}{\partial x} + \frac{\partial V_z}{\partial z}\Big) \tag{1}$$

here, x refers to the slope direction along the ground surface, and z to the downward direction perpendicular to the x axis. V_x and V_z are the x and z components of infiltration flow velocities, θ is the water content by volume, and t is the time; V_x and V_z are given according to Darcy's[1] law as:

$$V_x = -K_x\frac{\partial\phi}{\partial x}, \quad V_z = -K_z\frac{\partial\phi}{\partial z} \tag{2}$$

here ϕ and K_x, K_z refer to the total head and the hydraulic conductivity in the $x-$ and z directions, respectively. When (2) is substituted into (1), the left side of (1) is expressed by :

$$\frac{\partial\theta}{\partial t} = \frac{\partial\theta}{\partial\psi}\frac{\partial\psi}{\partial t} \tag{3}$$

and introducing the specific water capacity $C(\psi) = \partial\theta/\partial\psi$, (1) is expressed by:

$$C\frac{\partial\psi}{\partial t} = \frac{\partial}{\partial x}\Big(K_x\frac{\partial\phi}{\partial x}\Big) + \frac{\partial}{\partial z}\Big(K_z\frac{\partial\phi}{\partial z}\Big) \tag{4}$$

Using a slope gradient $=\alpha$, the total head ϕ is given by:

$$\phi = \psi - x\sin\alpha - z\cos\alpha \tag{5}$$

When ϕ is converted into ψ, (4) is expressed by:

$$C\frac{\partial\theta}{\partial t} = \frac{\partial}{\partial x}\{K_x\Big(\frac{\partial\psi}{\partial x} - \sin\alpha\Big)\} + \frac{\partial}{\partial z}\{K_z\Big(\frac{\partial\psi}{\partial z} - \cos\alpha\Big)\} \tag{6}$$

The relations between $\psi \sim \theta(\psi)$ and $\psi \sim K(\psi)$ are expressed by Tani's[1] equation:

$$\theta = (\theta_s - \theta_r)\Big(\frac{\psi}{\psi_0} + 1\Big)exp\Big(-\frac{\psi}{\psi_0}\Big) + \theta_r \tag{7 - 1}$$

$$K = K_s {S_e}^{\beta} \tag{7 - 2}$$

C in (6) can be given by the following equation, obtained by partially differentiating (7) with respect to ψ:

$$C(\psi) = -(\theta_s - \theta_r)\frac{\psi}{{\psi_0}^2}exp\Big(-\frac{\psi}{\psi_0}\Big), \quad [\psi = \psi'(\psi' \leq 0), \quad \psi = 0(\psi' > 0)] \tag{8}$$

S_e in (7) is the effective degree of saturation, defined as:

$$S_e = \frac{\theta - \theta_r}{\theta_s - \theta_r} \tag{9}$$

here θ_s is the saturated water content; θ_r the water content assuming little movable water; ψ_0 the pressure head to give the maximum specific water capacity C; K the saturated hydraulic conductivity: and β the gradient of unsaturated hydraulic conductivity in the unsaturated domain.

The boundary conditions are expressed with A, B, C, and D in Figure-2. The boundary at AB and BC have no flow passing then, through CD is the free outflow, and through AD is the condition that the rainfall is absorbed. Specifically, the boundary conditions are given by:

$$V_x = 0 \quad (\mathrm{AB}), \qquad V_z = 0 \quad (\mathrm{BC}), \qquad \frac{\partial V_x}{\partial x} = 0 \quad (\mathrm{CD}), \qquad V_z = r_*(t)\cos\alpha \quad (\mathrm{AD}) \tag{10}$$

where $r_*(t)$ is the infiltration rainfall·intensity, with,

$$\begin{cases} \psi \leq 0: & r_* = \mathit{rainfall\ intensity} = r(t) \\ \psi > 0: & r_* = \mathit{possible\ infiltration\ quantity\ into\ soil\ layer} \end{cases} \tag{11}$$

OVERLAND FLOW MODEL

Surface flow begins when the surface soil layer is saturated, i.e. when suction ψ is more than 0. However, underground flow infiltrated the soil layer when it was under saturated conditions. The calculation is conducted using the infiltration equations of Kavvas's study[3].

The rate of input to the surface flow ,q_o, can be calculated by the following equation.

$$q_o(t) = r(t) - K_z\Big(\frac{\partial \psi}{\partial z} - cos\alpha\Big) \tag{12}$$

In which $r(t)$ =total rainfall. The second term of the right hand side of equation (12) denotes the infiltration rate into the ground water flow.

However, in this study the rate of surface flow input $q_o(t)$ occurred when ψ became greater than 0. Rainfall went through the unsaturated surface at other times.

According to Kavvas[3], the momentum equation of surface flow is expressed by:

$$\frac{\partial h}{\partial t} + C_w \frac{\partial h}{\partial x} - q_o(t) = K_o \frac{\partial^2 h}{\partial x^2} \tag{13}$$

where h is the depth of flow, C_w and K_o are coefficients, given by the next following equations.

$$C_w = V_0(1+m) \tag{14}$$

$$K_o = \frac{V_o\ h\ j}{S_f} \tag{15}$$

where V_o is overland flow velocity, given by the next form.

$$V_o = D\ h^m {S_f}^j \tag{16}$$

Manning's equation ($m = 2/3, j = 1/2$) was adopted in this study. S_f in (15) was assumed to be equal to the water surface slope.

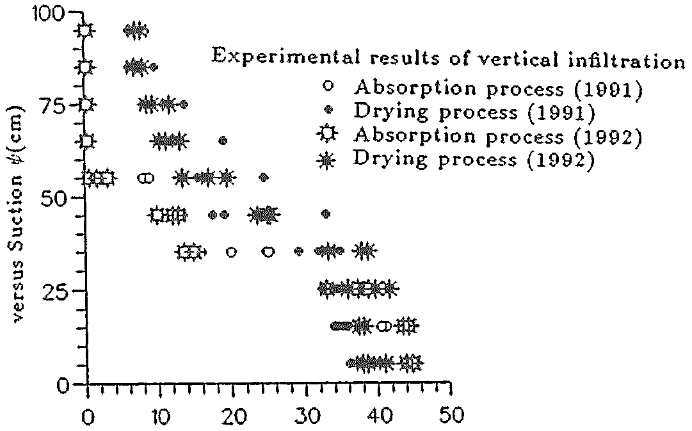

Figure-11 Water Content by Volume θ versus Suction ψ

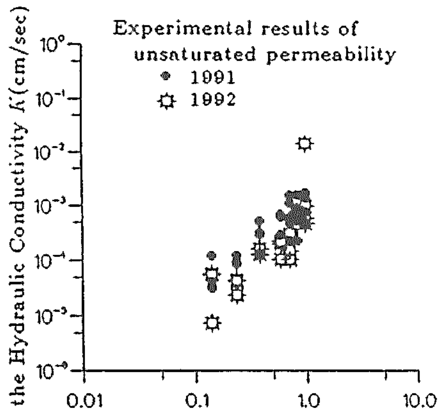

Figure-12 The Effective Degree of Saturation S_e versus the Hydraulic Conductivity K

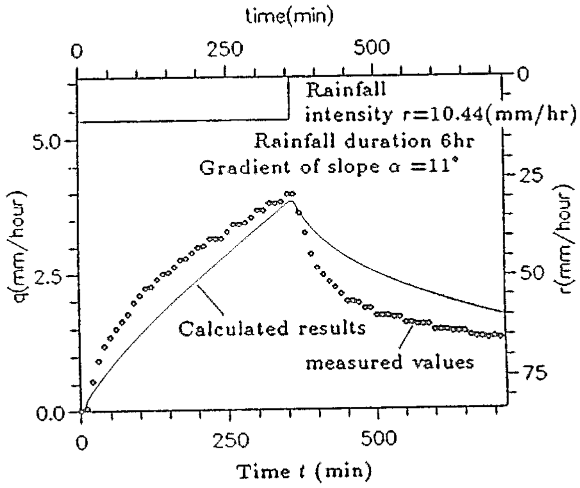

Figure-13 Changes in Rainfall Intensity r and Infiltration Runoff Discharge q_1(Run 1)

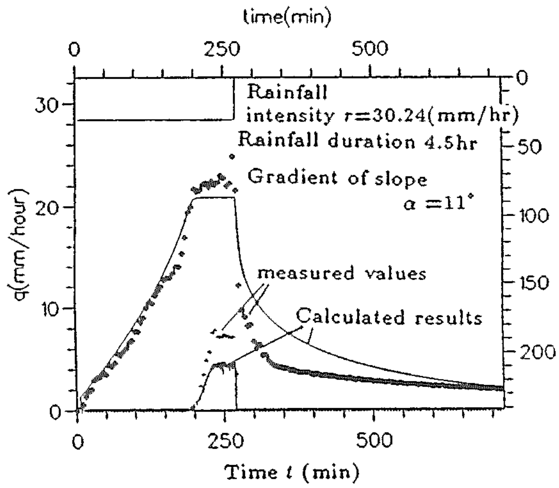

Figure-14 Changes in Rainfall Intensity r and Infiltration Runoff Discharge q_1 (Run 3)

Calculated results with the above model under the conditions of Run 1 and 3 are compared with the experimental results. In the calculation of Run 1 without surface runoff, it was assumed that all rainfall was infiltrated into the ground surface.

And in the calculation of Run 3, the surface runoff was assumed to be started when the rainfall intensity became greter than the possible infiltration rate.

To investigate the permeability characteristics of the experimental sand, vertical infiltration and unsaturated permeability tests were conducted.

Figure-11 shows the relation between the water content by volume θ obtained with the vertical infiltration test and the suction ψ .

In Run 1 $\theta_r = 0.04$, $\theta_s = 0.40$, and $\theta_0 = 29cm$ and in Run 3 $\theta_r = 0.07$, $\theta_s = 0.40$, and $\theta_0 = 29cm$ are the parameters in equation (7-1). The hydraulic conductivity at each suction value is given by the unsaturated permeability test and is converted into the relation between the effective saturation degree, S_e , and the hydraulic conductivity, K, by using the relation between θ and ψ in Figure-11, and the results are shown in Figure-12. The parameters in equation (7-2) used for the calculation are given as follows. For Run 1, β=2.00 and K=0.8 $\times$ 10^{-3}, and for ,Run 3 β=7.00 (cm/sec) and K =1.5 $\times$ 10^{-3} (cm/sec).

A numerical analysis was conducted under the experimental conditions of Run 1 and Run 3 using the above parameters. The experimental slope was divided into twenty and ten equal parts in the x and z directions, and the differentiation of (6) was made by the Crank Nicolson method.

Figure-13 Run 1 and figure-14 Run 3 indicate changes with time in rainfall intensity r and infiltration discharge q_1.

In figure-13 rainfall intensity r is equal to 10mm/hr. The dotted line represents measured values and the continuous line, the calculated results. The calculated results seem to be in good agreement with the measured ones at peak. Discharges are a little less at the beginning and a little greater under attenuate condition after the rain stops.

In Figure-14 rainfall intensity r is 30mm/hr. The dotted line represent measured and the continuous line, the calculated results. The calculated results at the peak point of the initial discharges are exactly the same as the measured values. However, under attenuated conditions, after the rain stops, the calculated results of discharges are greater. It is likely that choices of parameters or the soil layer affect the water holding capability. This problem should be further investigated. Figure-14 shows that the measured values of outbreak of the surface discharge are considerably less than the calculated results. This may mean that the established condition for outbreak of the surface flow should be reconsidered. Calculations are made for the outbreak of surface flow from saturated parts only when the soil layer surface is saturated.

SUMMARY

This study used a tilting flume filled with quartz sand and gravel.

Calculation results of the model with a rainfall intensity of 10 mm/hr are in agreement with the measured values but differ during the initial period and under attenuated conditions. A similar pattern of underground flow can be seen in the run where rainfall intensity is 30 mm/hr. However, surface flow is a little less since the surface flow is limited by the assumed boundary conditions.

Calculated results are generally in agreement with the measured values except when the parameter is different. A number of problems need to be investigated further.

REFERENCE

1) M HINO, T OHTA, K SUNADA, and K WATANABE: The Numerical Estimation of Floods, Morikita Shuppan Co., Ltd.p.120-128, 1989

2)Y ICHIKAWA and T YAMAMOTO: A Sutudy of Runoff Models Considering Ground Water Flows,Civil Engineering Society,Hokkaidou Bra.. Rep.. of Stu..,Vol.49 p519-524, 1993

3) Rao S.Govindaraju and M.L Kavvas: Dynamics of Moving Boundary Overland flow Over Infiltrating Surfaces at Hillslopes, Water resour.Res.,Vol.27 NO8.p1885-1898, 1991