Longitudinal Profile of Bed Elevation and Grain Size in Alluvial Rivers

by

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Abstract

A one dimensional model to predict the long term stability of channel bed elevation and grain size profile is proposed. The model is tested using the geometric conditions of the Ishikari River. By a series of calculations using different discharge profiles, it is shown that the two different characteristics of grain size distribution, with and without an abrupt change in the longitudinal direction, can be formed simply by changing the discharge scale. It is also shown that the formation of the bed elevation profile as well as the grain size profile can be calculated using time series of actual discharges over a wide range of magnitude.

Introduction

One of the characteristics of alluvial rivers is the downstream finning of river bed material. Fig.1 shows the downstream profile of bed elevation and mean diameter of bed material of (a)the Ishikari River, Hokkaido, Japan, and (b)the Toyohira River, one of the tributaries of the Ishikari River. The bed elevation in Fig. 1 is the cross sectionally averaged elevation calculated from the surveyed data. The grain size of the Ishikari River shows a gradual decrease in the downstream direction, but there is an abrupt change in grain size in the Toyohira River. The bed elevation changes gradually in both rivers. It was reported by Yamamoto *et al.*(1993) that most alluvial rivers in Japan can be classified into the same two groups as these two rivers, gradual change (Type-A) or abrupt change (Type-B) of the grain size. The same characteristic difference in grain size profiles can be found in other literature (see Parker(1991)).

A large number of studies have been made to understand the mechanism of downstream fining. Most previous studies have concluded that downstream fining is caused by selective transport of sediment and/or abrasion. Yamamoto *et al.*(1993) suggested that the abrupt change of grain size was caused by the rise of see water level in the past thousands of years. Parker(1991) proposed a model for the selective sorting and abrasion to predict the downstream fining of gravel-sand rivers. In the previous studies, the reason for the downstream profile of grain size having two different characteristics as Type-A and B has not been explained.

In this paper, an attempt is made to explain the two different characteristics of grain size distribution using a numerical model. A one dimensional flow equation is coupled with suspended and bed load transport equations for a mixture of grain sizes. Using several different discharges, the calculations are performed until the sediment transport reaches an equilibrium condition in the Ishikari River. It is shown that, even in the same river, the grain size profile can be changed drastically depending on the discharge. When the discharge is small, the longitudinal grain size profile show an abrupt change similar to Type-A rivers. This characteristic is not seen when the discharge is increased as similar to Type-B rivers. Abrasion is not taken into account in this study. However, the longitudinal grain size profile is well explained by the selective transport of sediment using the proposed model.

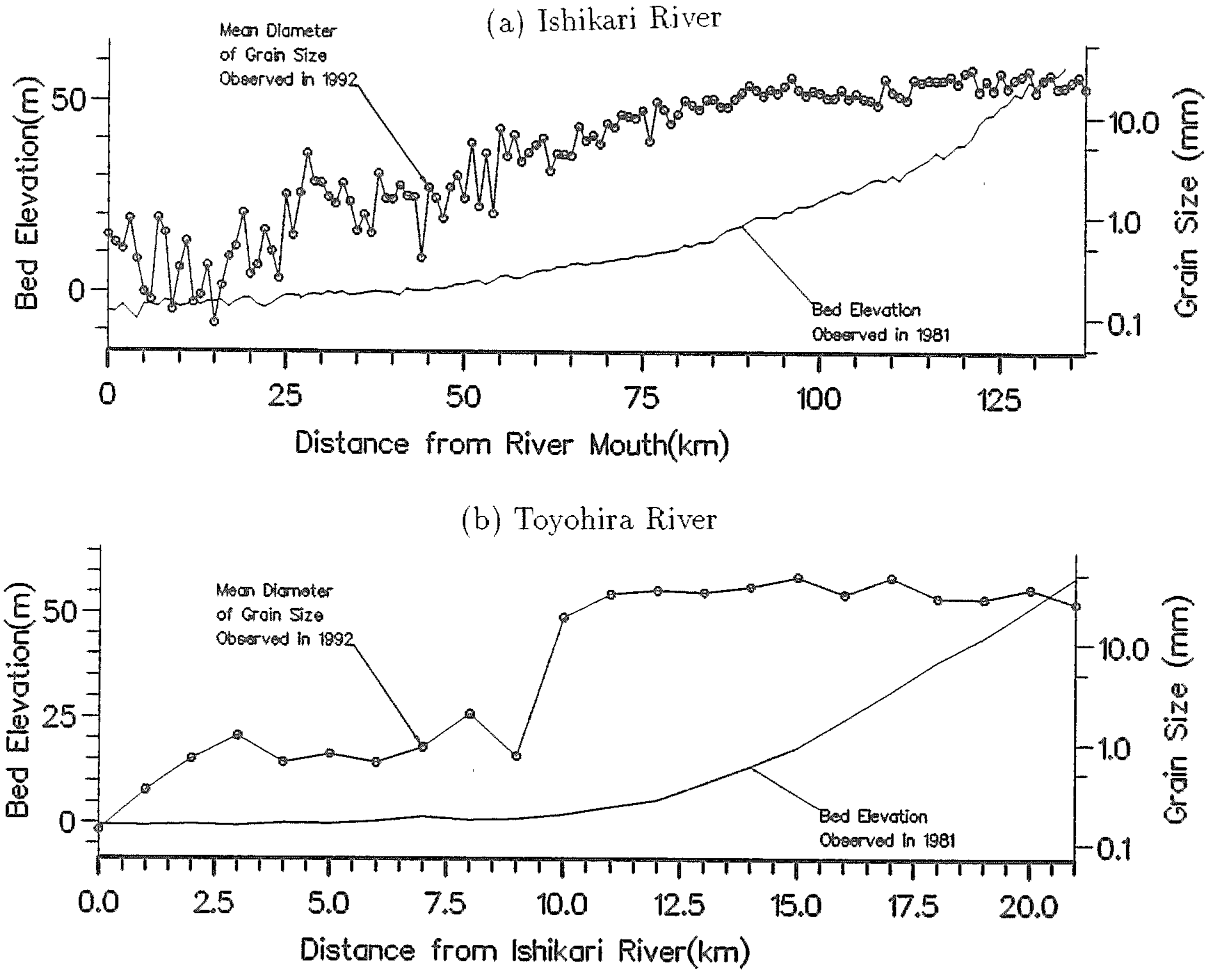

Figure 1. Longitudinal profile of bed elevation and mean diameter of bed material of (a) Ishikari River and (b) Toyohira River.

Basic Equations

A one dimensional momentum equation of flow is used to calculate the flow field.

$$\frac{\partial H}{\partial x} + \frac{\partial}{\partial x}\left(\frac{\alpha Q^2}{2gA^2}\right) + i_e = \frac{q_x Q}{gA^2} \tag{1}$$

Here, x = coordinate in direction of flow; Q = flow rate; g = acceleration due to gravity; H = water surface elevation; A = cross-section area of flow; q_x = lateral inflow rate per unit channel length; and i_e = energy gradient slope. Lateral inflow includes input from tributaries, overland flow and ground water flow. A Manning-Strickler type relationship is used for flow resistance [Kishi and Kuroki (1973)]:

$$\frac{u}{u_*} = 6.8\frac{h}{d_m}^{1/6} \tag{2}$$

in which u = mean flow velocity; u_* = shear velocity ($=\sqrt{ghi_e}$); h = flow depth; and d_m = mean diameter of bed material. Eq. 2 express the flow resistance when the river bed is flat or with bars. In the Ishikari river, it is reported by Kishi (1989) that the dunes are not fully developed but tend to collapse during the flood in the lower part of the river, and in other part of the river, bars dominate the flow resistance. Therefore, Eq. 2 should be valid for this study. The energy gradient i_e in Eq. 1 can be expressed using Eq. 2 and the continuity condition, $Q = Bu$ as

$$i_e = \frac{Q^2 d_m^{1/3}}{46.24 h^{10/3} g B^2} \tag{3}$$

in which B = channel width.

The following equation proposed by Ashida and Michiue(1972) is adopted to calculate the bed load transport rate.

$$\frac{q_{Bi}}{\sqrt{sgd_i^3}} = p_i 17 \tau'^{3/2}_{*i} \left(1 - \frac{\tau_{*ci}}{\tau_{*i}}\right)\left(1 - \frac{u_{*ci}}{u_*}\right) \tag{4}$$

Throughout this paper, subscript i denotes the grain size fraction of diameter d_i, and q_{Bi} = bed load transport rate per unit width; s= specific weight of sand grains in water; p_i = fraction of bed material of diameter d_i in the exchange layer; τ_{*i} = dimensionless bed shear stress ($=u_*^2/sgd_i$) in a mixture of bed materials; τ'_{*i} = dimensionless grain shear stress; τ_{*ci} = dimensionless critical bed shear stress ($=u_{*ci}{}^2/sgd_i$); and u_{*ci} = critical shear velocity. u_{*ci} is calculated by the following equation proposed originally by Egiazaroff(1965) and modified by Asada (1976).

$$\frac{u_{*ci}{}^2}{u_{*cm}{}^2} = \left[\frac{\log 23}{\log\left(21\frac{d_i}{d_m}+2\right)}\right]^2 \frac{d_i}{d_m} \tag{5}$$

in which u_{*cm} = critical shear velocity for the mean diameter of bed material, calculated by Iwagaki's (1956) formula.

The detachment rate of suspended sediment is calculated by the formula proposed by Itakura and Kishi (1980).

$$q_{sui} = p_i K\left(\alpha_* \frac{\rho_s - \rho}{\rho_s}\frac{gd_i}{u_*'}\Omega_i - w_{fi}\right) \tag{6}$$

$$\Omega_i = \frac{\tau'_{*i}}{B_{*i}} \frac{\int_{a'}^{\infty} \xi \frac{1}{\sqrt{\pi}} \exp(-\xi^2) d\xi}{\int_{a'}^{\infty} \frac{1}{\sqrt{\pi}} \exp(-\xi^2) d\xi} + \frac{\tau'_{*i}}{B_{*i}\eta_0} - 1 \tag{7}$$

in which q_{sui} = detachment rate of suspended sediment from unit area of bed surface; w_{fi} = fall velocity of sediment; $a' = B_{*i}/\tau'_{*i} - 1/\eta_0$; $\eta_0 = 0.5$; $\alpha_* = 0.14$; $K = 0.008$; ρ_s = density of sediment; ρ = density of water; and $\tau'_{*i} = u'^2_*/sgd_i$. w_{fi} is calculated by Rubey's formula (1933). B_{*i} is a coefficient to evaluate the lift force from the shear velocity. The following equation to calculate B_{*i} is proposed by Oki and Kuroki(1985) for the mixed grain size bed material.

$$B_{*i} = \xi_i B_{*0} \tag{8}$$

$$\xi_i = \frac{\tau_{*ci}}{\tau_{*ci0}} \tag{9}$$

where, $B_{*0} = 0.143$; $\tau_{*ci0} = u_{*ci0}{}^2/sgd_i$; and u_{*ci0} = critical shear velocity for the uniform grain size d_i, and which is calculated by Iwagaki's(1933) formula.

The continuity equation of suspended sediment is

$$\frac{\partial}{\partial t}(< c_i > h) + \frac{1}{B}\frac{\partial(Q < c_i >)}{\partial x} = q_{sui} - w_{fi}c_{bi} + \frac{q_x < c_{xi} >}{B} \tag{10}$$

in which t = time; c_{bi} = reference concentration of suspended sediment; $< c_i >$ = depth averaged concentration of suspended sediment; and $< c_{xi} >$ = depth averaged concentration of suspended sediment of the lateral inflow. The relation between $< c_i >$ and c_{bi} in Eq. 10 can be determined from the vertical distribution of suspended sediment concentration. The following exponential distribution for the suspended sediment is adopted.

$$c_i = c_{bi}\exp(-\beta\xi) \tag{11}$$

in which c_i = concentration of suspended sediment; $\beta = w_{fi}h/\varepsilon$; $\xi = z/h$; z = vertical coordinate; ε = depth averaged diffusion coefficient ($=\kappa u_* h/6$); and κ = Von Karman's constant (=0.4). $< c_i >$ can be derived by averaging Eq. 11 from the bed to water surface.

$$< c_i >= \frac{1}{h}\int_0^1 c_i d\xi = \frac{c_{bi}}{\beta}(1-\exp(-\beta)) \tag{12}$$

Eq. 12 gives the relation between $< c_i >$ and c_{bi} in Eq. 10.

The continuity equation of sediment transport for nonuniform bed material is expressed as

$$\delta\frac{\partial p_i}{\partial t} + p_i^*\frac{\partial \eta}{\partial t} + \frac{1}{1-\lambda}\left[\frac{1}{B}\frac{\partial(q_{Bi}B)}{\partial x} + q_{sui} - w_{fi}c_{bi} - \frac{q_x c_{Bxi}}{B}\right] = 0 \tag{13}$$

in which

$$p_i^* = p_i \quad ; \partial\eta/\partial t \geq 0$$
$$p_i^* = p_{io} \quad ; \partial\eta/\partial t < 0, \eta_0 \leq 0$$
$$p_i^* = p_{im} \quad ; \partial\eta/\partial t < 0, \eta_0 > 0$$

in which η = bed elevation: δ = thickness of exchange layer of bed, which is assumed to be equal to d_{90}; λ = porosity of bed material; c_{Bxi} = concentration of bed load in lateral inflow; η_0 = initial bed elevation of the calculation; and p_{im} = fraction of bed material of diameter d_i deposited at the site from previous calculations under conditions of agradation.

The equation of total sediment transport is expressed as

$$\frac{\partial \eta}{\partial t} + \frac{1}{1-\lambda}\left[\frac{1}{B}\frac{\partial \sum_i(q_{Bi}B)}{\partial x} + \sum_i(q_{sui} - w_{fi}c_{bi}) - \frac{\sum_i(q_x c_{Bxi})}{B}\right] = 0 \tag{14}$$

in which $\sum_i$ denotes the summation of values over every fraction i.

Lateral inflow discharge can be expressed as

$$q_x = \frac{\partial Q}{\partial x} \tag{15}$$

Assuming that the sediment concentration of the lateral inflow is equal to that of the main flow, the following relations are adopted.

$$< c_{xi} >=< c_i > \tag{16}$$

$$c_{Bxi} = \frac{q_{Bi}B}{Q} \tag{17}$$

In this study, since the flat-bed type resistance law is adopted as Eq. 2, the grain shear stress is assumed to be equal to total shear stress as

$$\tau'_{*i} = \tau_{*i} \tag{18}$$

$$u'_{*i} = u_{*i} \tag{19}$$

Equilibrium condition of sediment transport

It has often been discussed that an equilibrium bed elevation and grain size distribution of a river bed can be described by a characteristic discharge, such as bankfull discharge. In this section, the equilibrium state of bed elevation and grain size distribution is shown using the equations presented in the previous section. At an equilibrium state, the time dependent terms in Eq. 10 and Eq. 13 are dropped.

$$\frac{1}{B}\frac{\partial(Q < c_i >)}{\partial x} = q_{sui} - w_{fi}c_{bi} + \frac{q_x < c_{xi} >}{B} \tag{20}$$

$$\frac{1}{B}\frac{\partial(q_{Bi}B)}{\partial x} + q_{sui} - w_{fi}c_{bi} - \frac{q_x c_{Bxi}}{B} = 0 \tag{21}$$

Using Eqs. 20, 21, and 15, the following equation is derived.

$$\frac{\partial(Q < c_i >)}{\partial x} + \frac{\partial Q c_{Bi}}{\partial x} = (< c_{xi} > + c_{Bxi})\frac{\partial Q}{\partial x} \tag{22}$$

in which c_{Bi} = bed load concentration in the lateral inflow. When the sediment concentration in the lateral inflow is assumed to be equal to that of main flow as Eq. 16 and 17, Eq. 22 can be reduced to:

$$\frac{\partial}{\partial x}(< c_i > + c_{Bi}) = 0 \tag{23}$$

or

$$\frac{\partial c_{Ti}}{\partial x} = 0 \tag{24}$$

in which $c_{Ti} =< c_i > + c_{Bi}$

According to Eq. 24, the total sediment concentration (bed load plus suspended load) should be constant for each grain size fraction in every fraction of sand particles in the river from the downstream end to upstream end. The calculations shown in the following section are continued until the sediment transport satisfies Eq. 24.

Conditions of calculation

The Ishikari River was chosen as a test for this study because of the availability of survey data. Fig. 2 shows the observed cross sectionally averaged bed elevation of the Ishikari river in three different years in the past. Within these three years, the bed elevation is fairy constant and the following equation which approximates the observed bed elevation is used as a initial condition.

$$\eta = 4.764 \exp(0.0192 K_p) - 10 \tag{25}$$

in which η = meters above sea level and K_p = distance from river mouth in kilometers.

Fig. 3 shows the channel width obtained from three different years, and the following equation is used to calculate channel width.

$$B = 367 \exp(-0.01375 K_p), \quad K_p \leq 42 \tag{26}$$

$$B = 228 \exp(-0.00246 K_p), \quad K_p > 42 \tag{27}$$

in which B is in meters.

Fig. 4 shows d_{10}, d_{50} and d_{90} calculated from the observed grain size distribution. The following equations represent the longitudinal profile of d_{10}, d_{50} and d_{90}.

$$d_{10} = 0.205 \exp(0.0128 K_p) \tag{28}$$

$$d_{50} = 0.446 \exp(0.0269 K_p) \tag{29}$$

$$d_{90} = 1.260 \exp(0.0313 K_p) \tag{30}$$

in which units of d_{10}, d_{50} and d_{90} are in [mm]. The initial grain size fractions in and bellow the exchange layer are calculated from the values of d_{10}, d_{50} and d_{90} obtained from the above equations assuming a log-normal distribution.

Fig. 5 shows the discharge profile along the Ishikari River, in which circles denote the averaged annual maximum discharge at gage stations, and filled circles denote the yearly averaged discharge. The discharge data is based on the daily averaged discharge values observed for the past 30 years. Taking this discharge data into account, 6 discharge profiles as shown in Fig. 5 are chosen for the calculations. The form of the discharge profiles are as follows.

$$Q = Q_0 \exp(-0.00876 K_p) \tag{31}$$

in which Q_0 = discharge at river mouth in [m^3/sec.]. The values of Q_0 for the 6 discharge profiles are 400, 600, 1000, 2000, 4000 and 7000 m^3/sec., respectively.

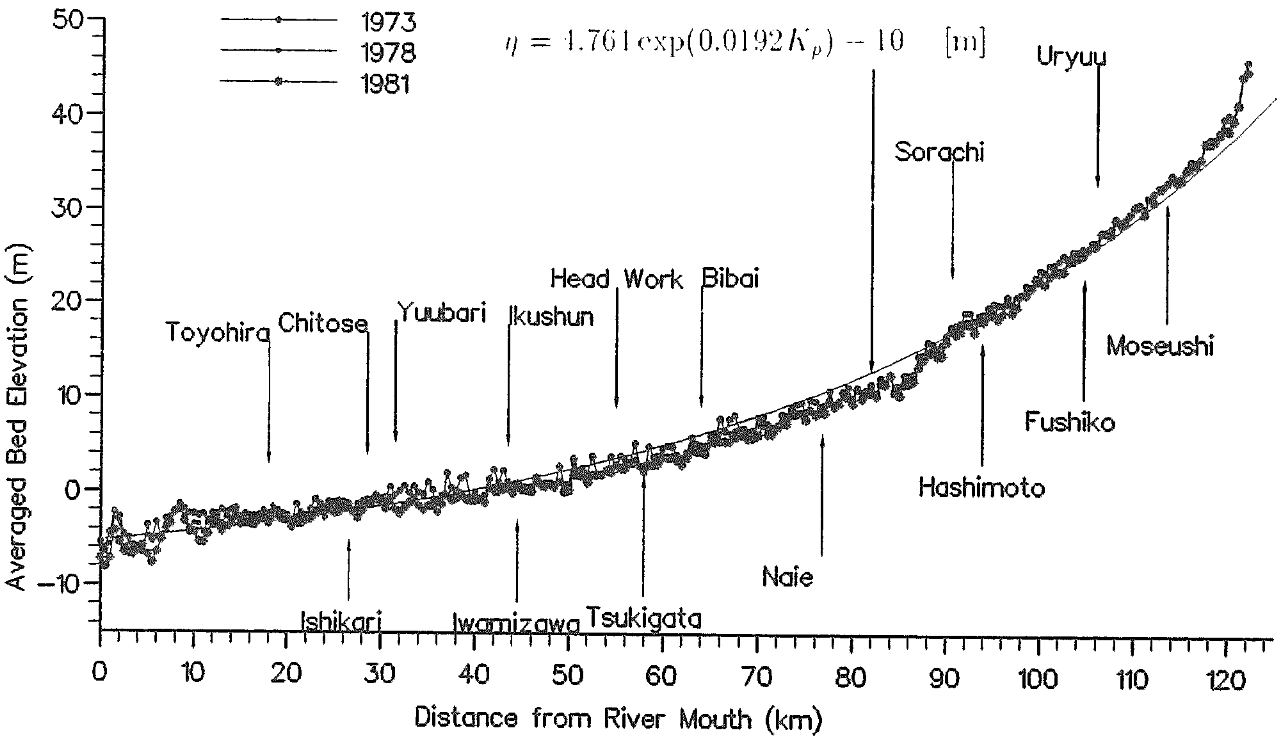

Figure 2. Longitudinal profile of bed elevation of the Ishikari River. Symbols are observed cross sectionally averaged values in three different years. Observed values are approximated by a smooth regression curve which is used for the initial condition of the calculations.

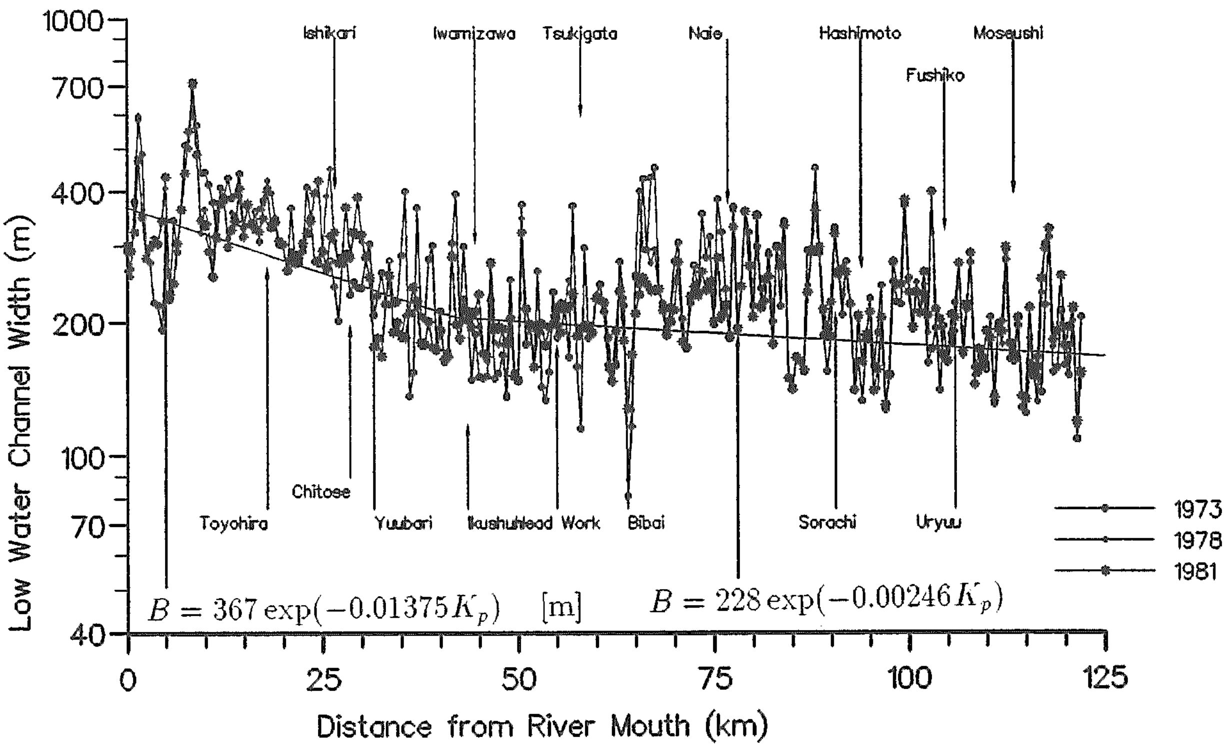

Figure 3. Longitudinal profile of channel width of the Ishikari River. Symbols are observed values in three different years. The smooth curve approximates the observed values used of channel width for the calculations.

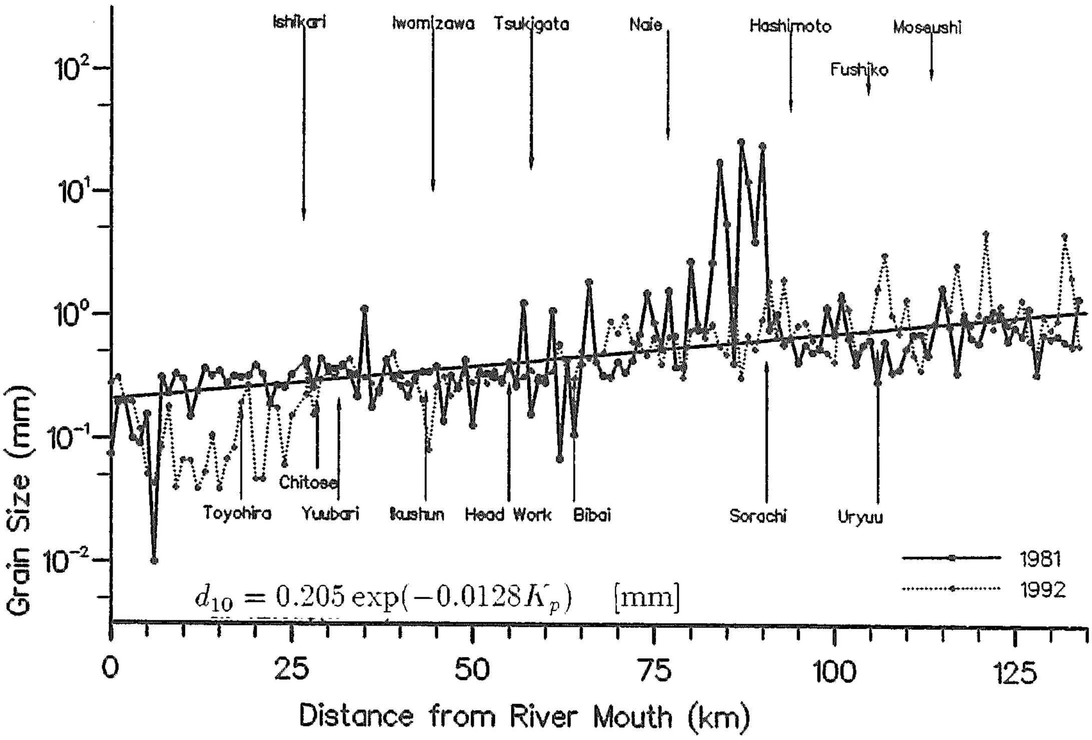

Figure 4(a). Longitudinal profile of d_{10}. Symbols are observed values in two different years. Lines are used as the initial conditions of the calculations.

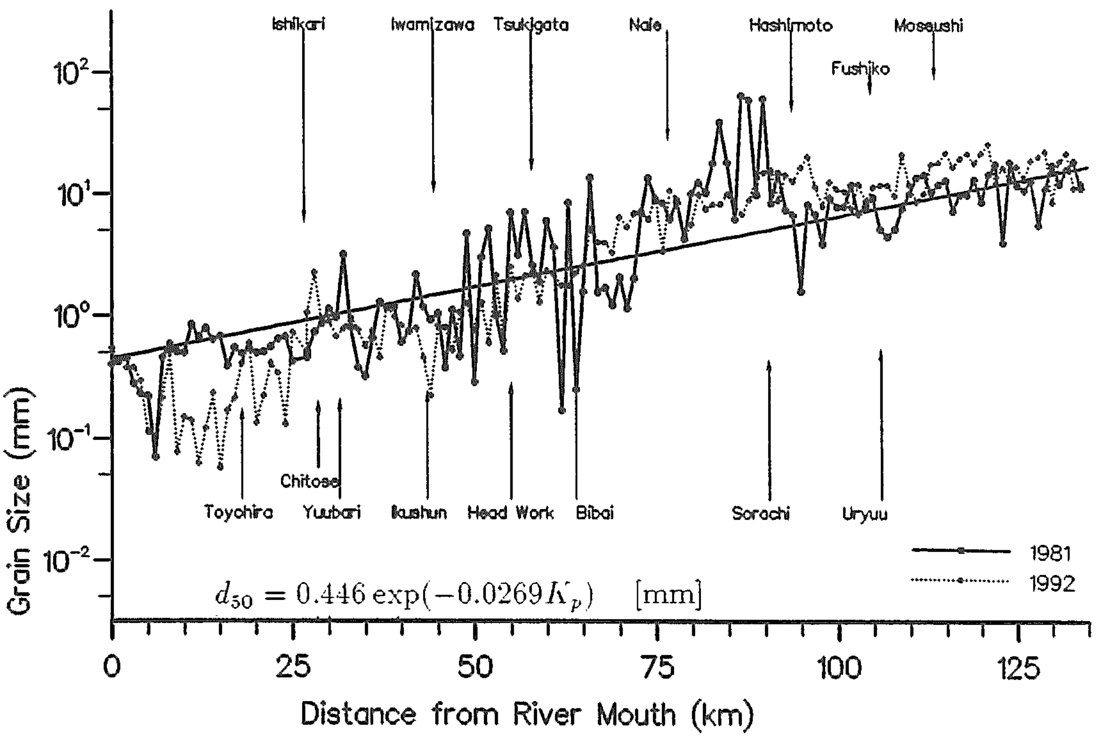

Figure 4(b). Longitudinal profile of d_{50}. Symbols are observed values in two different years. Lines are used as the initial conditions of the calculations.

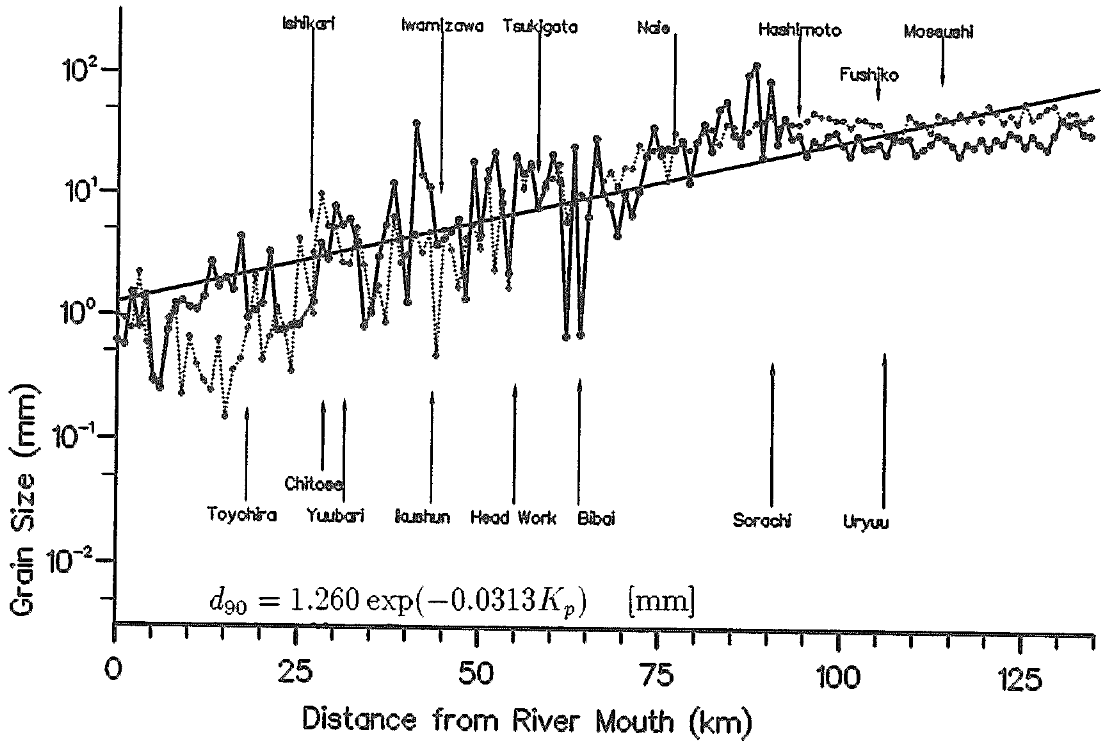

Figure 4(c). Longitudinal profile of d_{90}. Symbols are observed values in two different years. Lines are used as the initial conditions of the calculations.

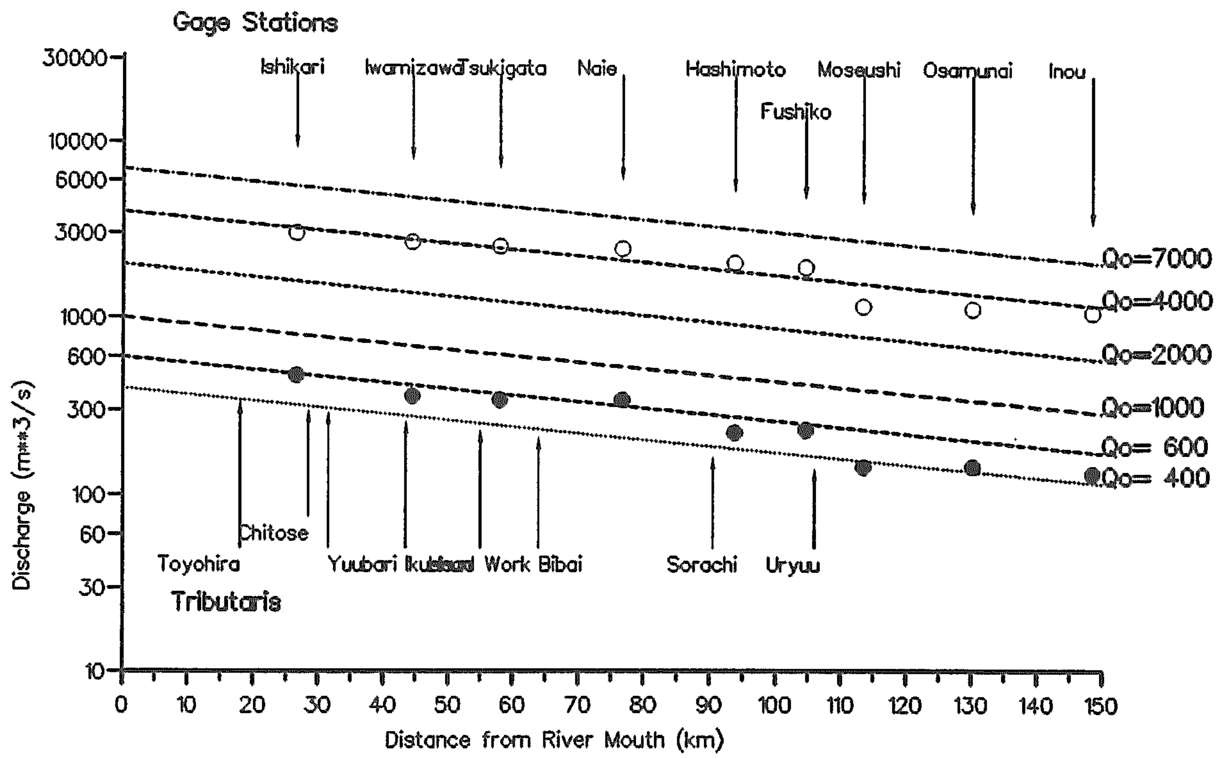

Figure 5. Longitudinal discharge profile of the Ishikari River. Circles are averaged annual maximum discharge and filled circles are yearly averaged discharge. The six curves are discharge profiles used for the calculation with constant discharge.

The water surface elevation at the river mouth is kept constant at 0.19m which is the average observed value for the past 30 years. Assuming the condition of dynamic equilibrium for sediment transport, the terms with partial derivative with respect to x in Eq. 11, 13 and 14 are set to be zero at the upstream end of the calculation. Calculations are performed by the finite difference method with 131 nodes in the x-directions. The grain sizes are divided into 21 diameters of d_i. Finite differential sizes of time(Δt) and the schematization are set according a criteria proposed by Kuroki *et al*(1980).

Using a given discharge profile, the flow field is calculated by Eq.1, bed load transport rate and detachment rate of suspended load is obtained from Eqs.4 and 6, respectively, and the suspended sediment concentration is calculated by Eq.10.

Calculated Results with constant discharge

Using the discharge profile of Eq. 31, calculations are performed until the sediment transport reaches an equilibrium which satisfies Eq. 24. Reaching equilibrium took 200, 100, 50, 10, 2 and 1 years when the values of Q_0 were 400, 600, 1000, 2000, 4000 and 7000 m^3/sec., respectively. Fig. 6 shows the calculated equilibrium profiles of the mean grain size diameter of the bed (exchange layer) d_m, compared with the initial profile of d_m (thick line) and the observed d_m (circles in Fig. 6) in 1992. In spite of the fact that the initial conditions and boundary conditions are identical in each of the calculations, the calculated bed grain size profiles show big differences depending on the discharge. When the discharge is small, only the fine materials are transported from the upstream region. These fine grain sizes are then deposited in the downstream region where the shear stress is small. The transported materials are relatively finer than the mean diameter of bed materials. Consequently, the grain size distribution of bed material becomes coarser in the upstream region and vise versa in the downstream region which forms a bimodelized grain size profile as similar to that of Type-A rivers. In contrast, when the discharge is increased, since the bed shear stress is large enough to transport most of the grain size in the bed, they are transported from the upstream and are not deposited in the downstream region, and thus the grain size profile becomes relatively smooth and the bimodelization is not seen, similar to that of Type-B rivers. The calculated results with smaller discharge, Q_0 = 400 or 600 [m^3/s], agree with the actual observed profile of bed material.

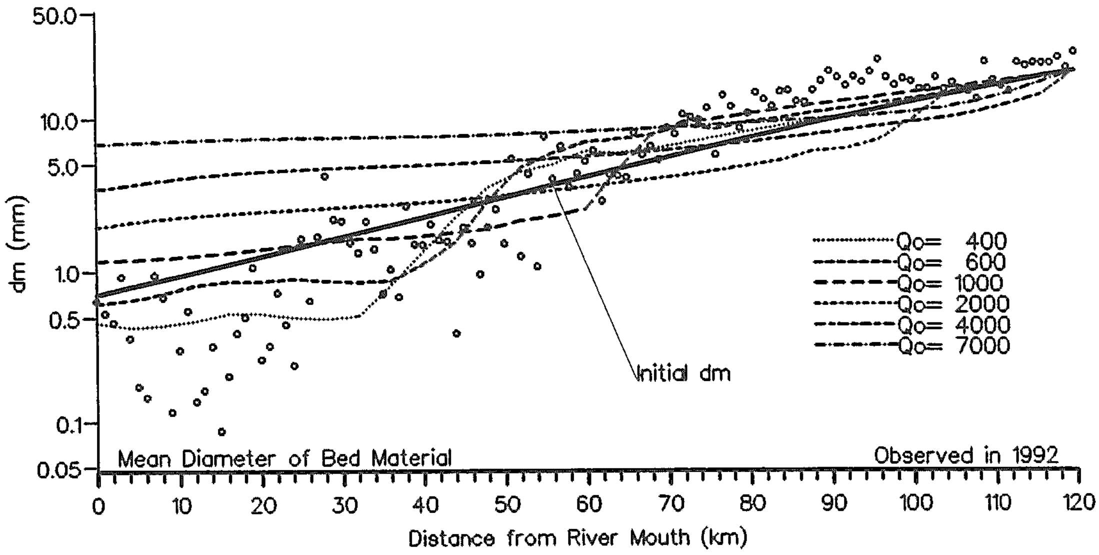

Figure 6. Calculated equilibrium profiles of mean grain diameter of bed material compared with initial profile (thick line) and observed values (circles).

Fig. 7 shows the equilibrium bed elevation profiles calculated using the 6 discharge profiles. The initial and observed profiles are included in this figure for comparison. There is no clear difference between the bed elevation profiles for different discharges, and all the calculated bed profiles are smoothly decreasing from upstream to downstream, similar to the observed bed profile. A tendency of deposition is seen in the downstream region when the discharge is small and is caused by the deposition of fine materials. Fig. 8 is the expanded version of

Fig. 7 in the downstream region. The agreement with the observed bed elevation is better when the discharge is relatively large (Q_0 = 2000 or 7000 [m^3/s]). Thus, the observed longitudinal grain size profile is well produced by small discharges. Whereas, the observed longitudinal elevation is well produced by large discharges.

The actual profiles of grain size and elevation are not formed by constant discharge as in the above calculations. However, these calculations show that the discharge profile can drastically change the bed material profile.

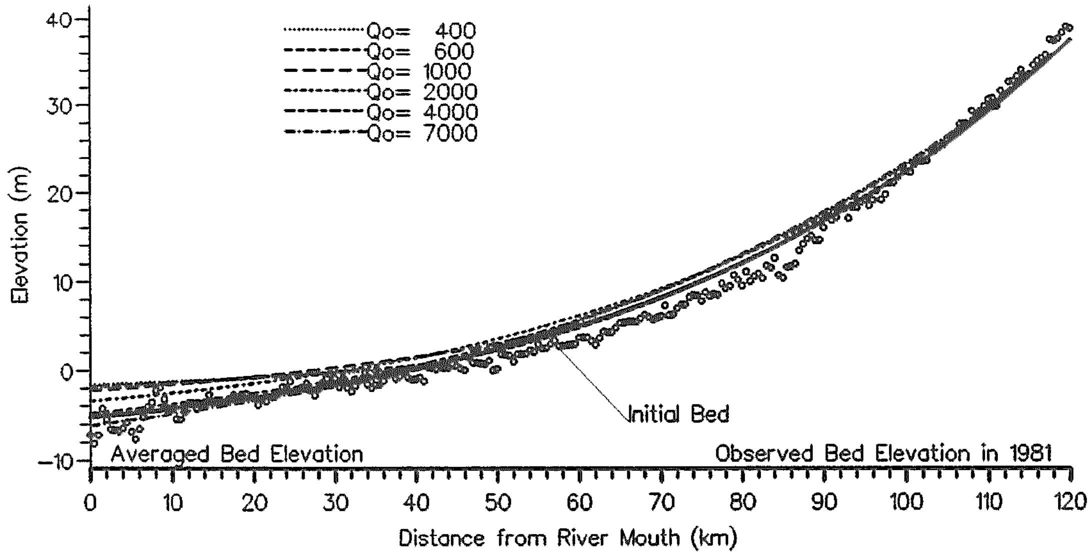

Figure 7. Calculated equilibrium profiles of bed elevation compared with initial profile (thick line) and observed values (circles).

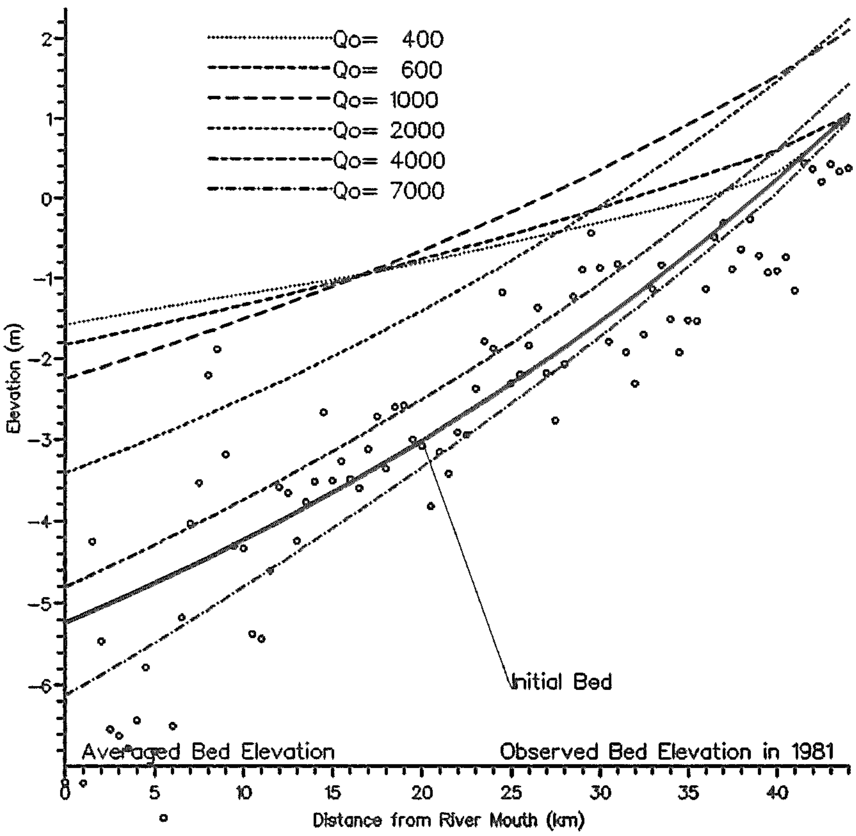

Figure 8. Calculated equilibrium profiles of bed elevation compared with initial profile (thick line) and observed values (circles).

Calculation with time series of discharge

In order to realistically produce the behavior of bed elevation and grain size, a series of daily discharge profiles from the past 10 years was used repeatedly in a series of the calculations. Fig. 9 shows the observed daily averaged discharge data between 1975 and 1984 at the mouth of the Ishikari River. Discharge plotted in Fig. 9 was used for Q_0 of Eq. 31. The 10 years discharge was used 5 times to simulate the long term of 50 years bed and grain size formation. Other conditions were kept identical to those of the calculations in the previous section.

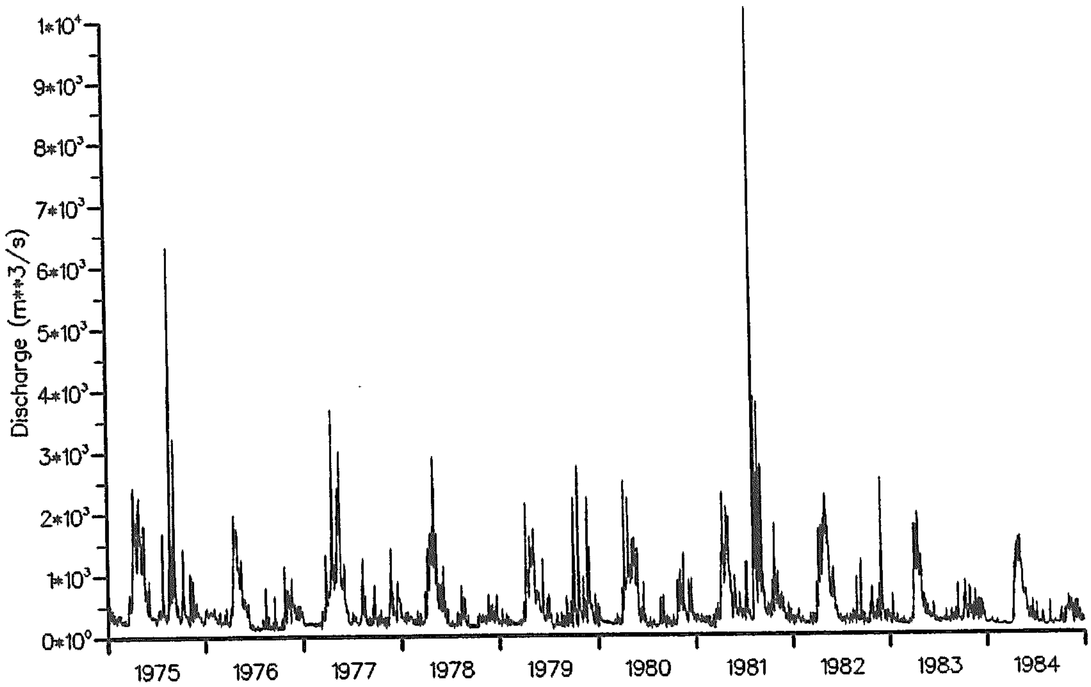

Figure 9. Observed daily averaged discharge data from 1975 to 1984 at the river mouth of the Ishikari River.

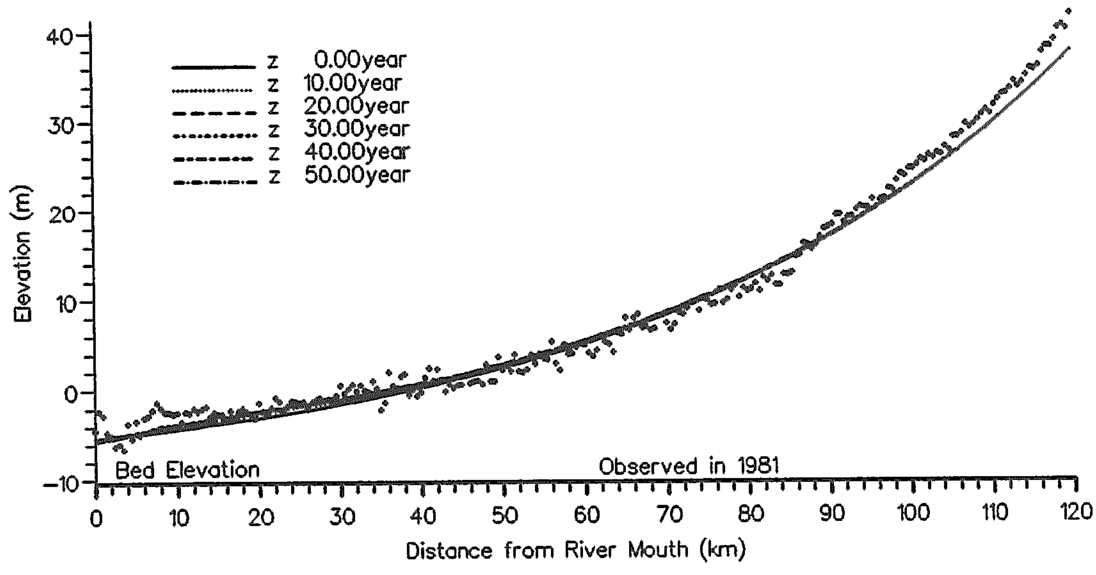

Figure 10. Longitudinal bed elevation profiles calculated using a series of recorded discharge compared with observed values.

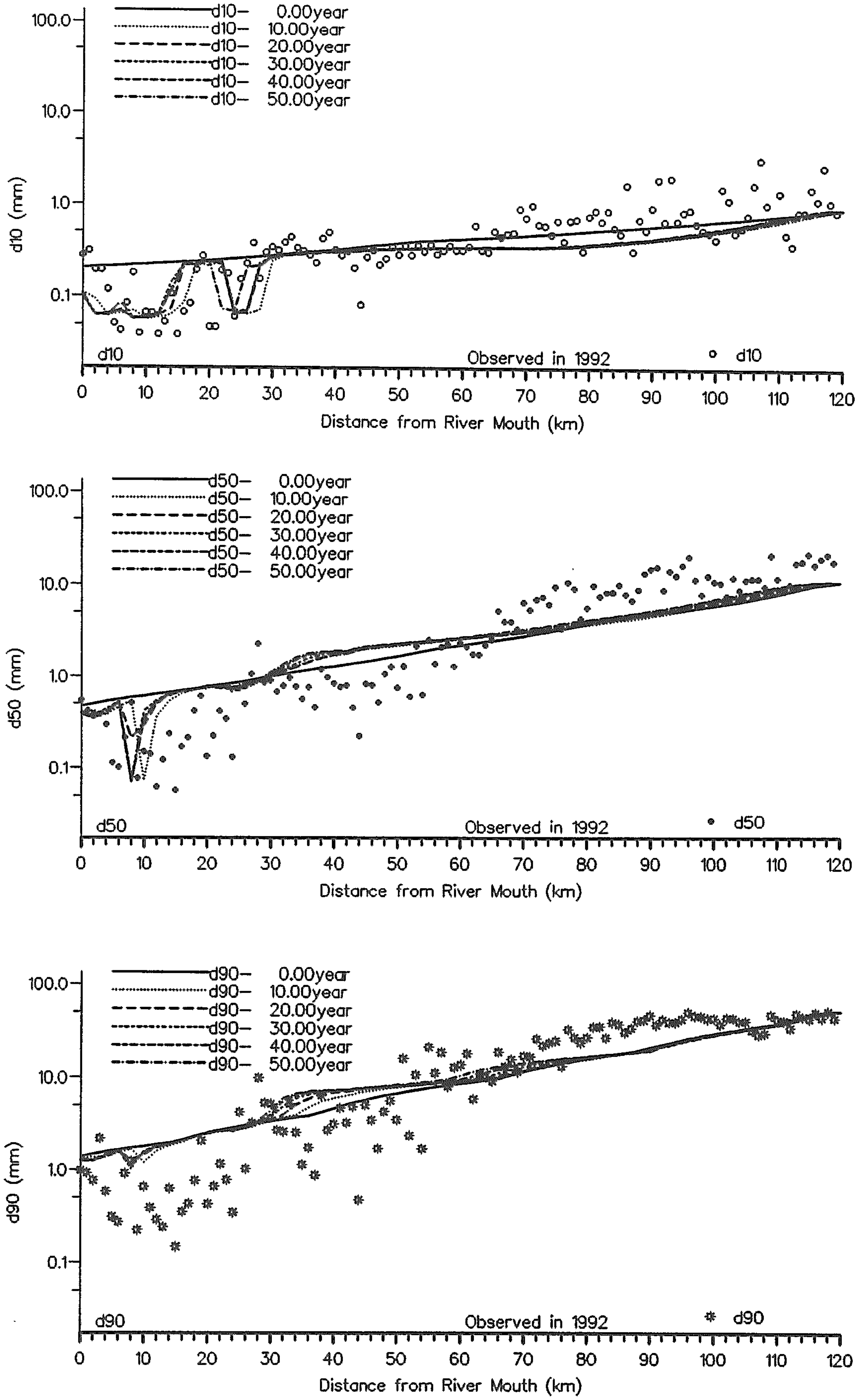

Figure 11. Longitudinal profiles of (a) d_{10}, (b) d_{50} and (c) d_{90} calculated using a series of recorded discharge compared with observed values.

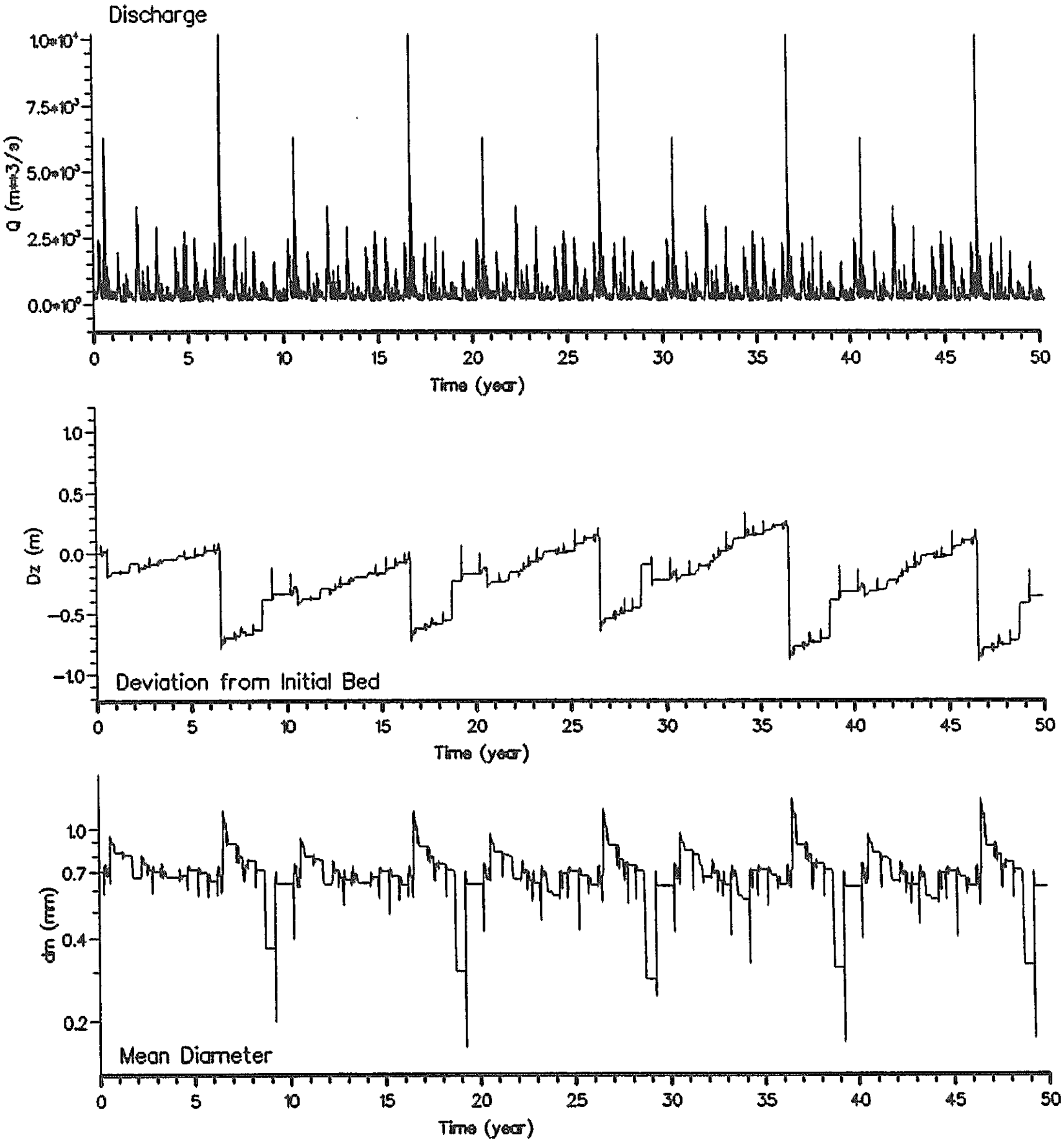

Figure 12. Calculated time evolution of bed elevation and mean diameter of bed material at the river mouth.

Fig. 10, and Fig. 11 show the change of bed elevation and bed grain size profiles (d_{10}, d_{50} and d_{90}), respectively. Bed elevation and bed grain size are very stable and agree well with the observed values. The calculated profile of d_{10} [Fig. 11(a)] becomes much finer in the downstream region. This is caused by the deposition of fine sediment transported from the upstream region, which is also seen in the observed data.

Fig. 12 shows the time changes of bed elevation and mean diameter d_m at the river mouth during the 50 year calculation, compared with time series of discharge. The bed is only eroded during the big flood which occurred in ten year series, and gradual deposition after the flood until the next flood ten years later. The grain size behavior dose not as clearly correspond to the discharge as dose the bed elevation.

It seems that the bed and grain size profile of the Ishikari river can only formed by continuous discharge profiles which include various sizes of floods as the calculation shown in this section.

Conclusion

A one dimensional model to predict the long term stability of channel bed elevation and grain size profile is proposed. The model was tested using the geometric condition of the Ishikari River. By a series of calculations using different discharge profiles, it was shown that the two different characteristics of longitudinal grain size distributions can be formed simply due to the discharge scale. When the discharge is relatively small, the grain size profile is bimodelized with an abrupt change, and when the discharge is relatively large, the grain size profile is relatively smooth. The two different characteristics of grain size profiles existing in natural rivers can be explained by the proposed model. However, it was not possible to predict both the bed elevation profile and the grain size profile with a single constant, "characteristic" discharge. This seems to be only possible by the combination of a wide range of discharge scales as actually occurs in real rivers.

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